MIP Insights

The newsletter of the Mixed-Integer Programming Society

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The 2023 MIP Workshop

The 2023 MIP Workshop was held on **May 22-25** at the **University of Southern California**. This was a special year for MIP: it marked the 20th anniversary of what has become one of the most important events in the integer programming and discrete optimization community. The workshop had 22 invited experts from around the world presenting state-of-the-art research in diverse areas, such as the theory of integer programming, combinatorial optimization, non-linear optimization, and applications, among others. As customary, MIP had a strong presence of junior researchers among its presenters (63%), and, as part of steady efforts from the community to increase women's representation, MIP had its largest ever women presence among its speakers (41%).

In addition to the invited presentations, the workshop hosted two competitions: the now-traditional **poster competition** and the second annual computational competition. Selected from a large competitive pool of applicants, 30 posters were presented in person at the conference. In total, four awards were given this year to Kristin Braun (Fraunhofer Institute for Integrated Circuits, Honorable Mention), Angela Morrison (University of Colorado Denver, Honorable Mention), and Noah Weninger (University of Waterloo, Most Popular Poster and Best Poster). For the **computational competition**, the topic of its second edition was *MIP Reoptimization*, where participants were asked to design methods for reusing information from one MIP to solve a similar one. In this edition, two awards were given: Paul Strang et al. (ISAE-SUPAERO, Honorable Mention) and Krunal Patel (Polytechnique Montréal, Competition Winner). The winner of the computational competition will receive an expedited review process in Mathematical Programming Computation.

As the cherry on top, to celebrate the 20th anniversary, we had a special contribution from some of the first organizers of the MIP series, who are still highly active members of our community. The 2023 organizing committee interviewed them to hear their thoughts about many aspects of the MIP workshop: its beginnings, how it has changed, what are their best memories, etc. We invite everyone to visit https://www.mixedinteger.org/2023/ to see the list of presenters, slides, more event photos, and interview videos.

The 2024 MIP Workshop

The 2024 Mixed-Integer Programming Workshop will be held on **June 3 - 6, 2024**, at the **University of Kentucky**. The workshop will cover a wide array of topics, including theory of integer programming, combinatorial optimization, and applications, presented by 20 experts from around the world.

In addition to the invited presentations, we are fortunate to have additional events at this year's workshop:

1. The workshop will host the **Third Annual Computational Competition**. The topic for this year's competition will be *MIP Presolve*. Please see here for more details. High-quality submissions to the computational competition will receive an expedited review process in *Mathematical Programming Computation*.
2. The workshop will continue the tradition of having a **poster session**. A call for posters will be made later this year, so keep an eye out! As done in the past, student finalists for the competition will compete for the workshop's best poster award.
3. The workshop this year will be preceded by one of the First **MIP summer schools**! The summer school will take place on June 2, 2024, and the speakers for this year's summer school include Santanu S. Dey, Robert Hildebrand, and Jean-Philippe Richard.

Please keep an eye out for more information by visiting the website https://www.mixedinteger.org/2024/index.html.

See you in Lexington!

Call for locations for MIP 2025

If you are interested in hosting MIP2025, then please send an email to the current program committee at joseph.paat@sauder.ubc.ca. We kindly ask you state your interest by February 1, 2024, at which time the program committee will start to evaluate potential locations. The committee will continue to evaluate potential locations until an appropriate one is chosen.

MIP International Workshop in Mumbai

The inaugural **MIP International Workshop** is being organised at **IIT Bombay, Mumbai, India** on **Dec 2 - 6, 2024**. The MIP International Workshop aim is to facilitate growth of discrete optimization research and research collaborations across the globe. The event is specifically designed to provide ample time for discussion and interaction between the participants. The workshop will consist of a single track of invited talks and a poster session. Thanks to the generous support by the Government of India, registration and stay are covered, and limited travel support is available for students and postdocs who present posters. More details about the event will be available soon on the MIP website.

Organizing Committee: Avinash Bhardwaj (IIT Bombay) and Vishnu Narayanan (IIT Bombay)

Program Committee: Mathieu Van Vyve (Chair, UC Louvain), Kavitha Telikepalli (Tata Institute of Fundamental Research), Diego Moran (Rensselaer Polytechnic Institute), Chen Chen (Ohio State University), and Sriram Sankaranarayanan (Indian Institute of Management Ahmedabad).

Discrete Optimization Talks (DOTs)

The Mixed-Integer Programming Society supports **Discrete Optimization Talks (DOTs)**, a virtual seminar series on all aspects of integer and combinatorial optimization. Visit talks.discreteopt.com to find information on the Fall 2023 season of DOTs and view recordings of previous talks. To receive the link to participate, join the mailing list and add "lists@mixedinteger.org" to your approved addresses. If you are interested in giving a DOT, let us know.

Call for session chairs at ISMP 2024

The 25th **International Symposium on Mathematical Programming** (ISMP) will be taking place in **Montreal, July 21-26 2024**. Details are here. The scientific committee in charge of the *Mixed Integer Linear Programming* stream of the *Discrete Optimization* cluster solicits all researchers interested in organizing a session on topics in MIP to write to MIP.ISMP.2024@gmail.com with a tentative session title. Each session will have a duration of two hours and will be composed of four 30-minute talks, including 5 minutes for questions and for changing speakers. A session chair can be one of the speakers in their session, but they cannot speak in a different session. Moreover, each attendee can give at most one talk at ISMP.

In 2022, the Mixed-Integer Programming Society (MIPS) was established as a technical section of the Mathematical Optimization Society. In the MIP Insights *newsletter, we announce important news for the community, promote events supported by the society, and provide reports on recently concluded activities. We also host expository presentations of relevant results in the area. Avinash Bhardwaj, Yuri Faenza, Aleksandr M. Kazachkov, Gonzalo Muñoz, Vishnu Narayanan, Joseph Paat, and Stefan Weltge contributed to this issue.*

A chat with two Integer Programmers co-awardees of the 2023 Gödel Prize

Sam Fiorini (left) and Thomas Rothvoss at the Twelfth Cargese-Porquerolles Workshop on Combinatorial Optimization in September 2023.

The **Gödel Prize** is awarded yearly by the Association for Computing Machinery (ACM) Special Interest Group on Algorithms and Computational Theory (SIGACT) and the European Association for Theoretical Computer Science (EATCS) for outstanding papers in the field of theoretical computer science. It is one of the most prestigious awards in the field. This year's award is shared by two papers:

> Fiorini, Samuel; Massar, Serge; Pokutta, Sebastian; Tiwary, Hans Raj; de Wolf, Ronald (2015). **Exponential Lower Bounds for Polytopes in Combinatorial Optimization**. Journal of the ACM. 62 (2): 17:1–17:23.

> Rothvoss, Thomas (2017). **The Matching Polytope has Exponential Extension Complexity**. Journal of the ACM. 64 (6): 41:1–41:19.

This is an incredible recognition for the authors, and for our field more generally. On the occasion of the *Twelfth Cargese-Porquerolles Workshop on Combinatorial Optimization*, we talked with two of the co-recipients of this award – **Samuel Fiorini** (speaker at MIP 2022) and **Thomas Rothvoss** (speaker at MIP 2013) – about the genesis of their awarded works, their impacts on the authors' careers, and interesting open questions in the area.

Sam, Thomas, congratulations on your award and thank you for agreeing to chat with us. Let's maybe start by describing the contributions that led to the award. Sam, would you please describe yours?

Sam: Many of the attempts to prove P=NP rely on the existence of Linear Programs (LP) that solve NP-Complete problems. In particular, there were claims that there exist "small" LPs describing the TSP polytope, i.e., the convex hull of all solutions to instances of the Traveling Salesman Problem, which is a famous NP-Complete problem *[The Traveling Salesman Problem aims at finding the smallest tour visiting a set of n cities.]* Such "small" LPs are called (*linear*) *compact extended formulations*. If those claims were true, then one would be able to solve any instance of the TSP by employing efficient algorithms that solve LPs. Hence, we could deduce that P=NP. More than 20 years prior to our work, Yannakakis [13] showed however that no *symmetric* compact extended formulation exists for the TSP polytope. We showed more generally that no compact extended formulation - symmetric or not - can describe the TSP polytope. So those approaches to the P vs. NP questions were bound to fail.

What was the status of the field when you started working on the problem?

Sam: The contribution by Yannakakis was highly regarded in the theoretical computer science community, but for many years nobody could improve over his results. In the MIP community, his work was not so well-known. Around 2010, researchers in the MIP community also started working in this area. In particular, the survey by Conforti, Cornuéjols, and Zambelli [2] appeared, and the paper [7] by Kaibel, Pashkovich, and Theis showed that symmetry matters for some problems – that is, there are problems that admit no symmetric compact extended formulations, but do admit non-symmetric compact extended formulations. In early 2010, at the *Aussois Workshop in Combinatorial Optimization*, together with Gianpaolo Oriolo, Gautier Stauffer, and Paolo Ventura, we decided we wanted to learn about those results. So we organized the first *Cargese Workshop in Combinatorial Optimization* and invited Francisco Barahona, Michele Conforti, Michel Goemans, and Volker Kaibel as main speakers to lecture about their and other results in the area.

Several open problems were posed at that workshop, right?

Sam: Indeed. For instance, whether there exists *any* polytope with 0/1 vertices that cannot be described with a compact extended formulation. This is an even more basic question than the one for the TSP polytope. Thomas settled this question briefly afterwards [12].

Thomas: Yes, I also was at the workshop and when I heard this question I immediately connected it to a similar result on *Circuit Complexity*. We do not know any explicit function with high circuit complexity, but by a counting argument one can show that there must be *some*. That's actually an easy proof, taught in undergraduate classes in computer science. So I thought that maybe a similar approach could work...and it did.

Back to Sam. Can you tell us how you started working on the problem that led to your awarded paper?

Sam: I was mostly driven by curiosity, I wanted to push forward the results by Yannakakis. Among other contributions, he showed a relationship between extended formulations and *Communication Complexity*, which is a measure of complexity of matrices very popular in computer science. In particular, he showed that *certain* extended formulations for a polytope P can be obtained starting from communication protocols computing a matrix related to P, called *slack matrix*. So my first goal was to understand this relationship further and identify the communication model that describes *every* extended formulation. This we achieved together with Yuri Faenza, Roland Grappe, and Hans Tiwary when I visited the University of Padova [4].

Then, together with Sebastian Pokutta and Hans Tiwary, we started thinking how all those results could be extended to *Semidefinite* extended formulations. There is a relationship between such formulations and *Quantum* communication complexity, but unfortunately I did not know much about Quantum back then. So I looked for some expert in the area once back at the Université libre de Bruxelles, where I work, and I found one while playing with one of my kids!

This sounds like an interesting coincidence. Tell us more about it.

Sam: I was playing soccer with my oldest son in a park, and Serge Massar was also playing with his son there. I knew him as an expert in Quantum *Information Theory*, so I took the opportunity to approach him and mention him the problem. Serge included in the discussion Ronald de Wolf, who is also an expert in Quantum communication. Our goal at that point was to find any matrix that could be computed efficiently by a Quantum communication protocol but not by a classical one. We found such a matrix, but there was a slight problem: it was not a *slack* matrix, hence it did not correspond to a polytope. However, from the work of Pashkovich [10], we knew that the matrix corresponded to a polyhedral pair. Basically, we had two polytopes, one contained in the other. Using a result of Razborov [11], we knew that none admits a compact extended formulation. But we did not know which polytopes we were dealing with. It just so happened that the smaller one was the correlation polytope.

So your original goal was not to show that the TSP polytope does not have a compact extended formulation.

Sam: No, it was not. But then we understood that we could employ the results obtained so far to settle the open problem on the TSP polytope. It took us a while to realize we had solved this open question, actually. We were all surprised, I guess.

On to Thomas. Can you tell us about the result in your awarded paper?

Thomas: The matching polytope is the convex hull of matchings in a complete graph. Thanks to Edmonds' work [3], we have an explicit inequality description for this polytope, that however has exponentially many constraints. So the question is: is there a compact extended formulation for the matching polytope? I showed that this question has a negative answer.

How did you start working on the problem?

Thomas: When I saw the paper by Sam and co-authors on *arXiv* I was stunned, since I did not think one could come up with explicit 0/1 polytopes that do not have a compact extended formulation. People started asking whether one can prove similar bounds for the matching polytope. Unlike the TSP, we know how to optimize a linear function over the matching polytope in polynomial time, so it was unclear what the answer should be. Also, the technique of *rectangle covering* that it was used for the TSP polytope cannot work for the matching polytope, and this called for new ideas.

So I went back to the paper by Razborov, read it multiple times, and tried to make it work for the matching polytope. I could not use it as a black-box result, but I was able to mimic its approach and apply it to the matching polytope. For many weeks, it felt like I had a blanket that was too short: I pulled it on one side, and then on the opposite side, but never succeeded in simultaneously obtaining all the properties I needed for the proof, until I eventually could.

What was it like to work on this problem?

Thomas: I dedicated all of my working time to this project, for a period between 4 and 6 months when I was a post-doc with Michel Goemans at MIT. At that time, I had already secured a job at the University of Washington, so I had a lot of uninterrupted time to think about the problem. This helped a lot.

What did you do when you completed the project?

Thomas: The final proof is quite technical, so I had to read it multiple times to convince myself it was correct. I sent it to Sam, who gave good feedback and believed it was correct as well, which was comforting.

Sam, Thomas: how did these results affect your career?

Sam: They had a huge impact for me. I got a *Best Paper Award* at STOC 2012, and there were enough interesting open problems in the area that I got an *ERC Grant* from the European Union to study them. This grant freed a lot of my time, letting me work on more problems.

Thomas: It had a profound influence of my career. Awards (*Best Paper Award* at STOC 2014, *Fulkerson Prize*, ...), large grants, and it changed how people treated me.

Did you tell your families about your papers and awards? Did you explain your results to them?

Sam: They were impressed by the prize, but I did not tell them about the result.

Thomas: My wife knows it is a non-existential result, and she consequently makes fun of how "useless" it is.

To conclude this interview, what are the important open problems in the field, in your opinion?

Sam: There is still some gap between the lower bound on the *size*, i.e., the number of inequalities, of the smallest extended formulation for generic $0/1$ polytopes obtained in the paper by Thomas [12] via a non-constructive proof, and lower bounds for explicit classes of polytopes. The work by Göös, Jain, and Watson narrowed this gap [6], but closing it completely would be nice.

Another interesting question deals with *approximate* extended formulations, i.e., extended formulations for any polytope that is sandwiched between a given polytope P and an appropriate scaling of P. Although those extended formulations seem easier to obtain than exact extended formulations (since they do not have to project exactly to P), it is still reasonable that for a wide classes of polytopes, compact approximate extended formulations do not exist. So far we only have negative results about constant scaling, while we would like to show that for interesting classes of polytopes, compact approximate formulations with a scaling factor that grows with the dimension do not exist.

A third stream of research deals with lower bounds on the size of semidefinite extended formulations, also in connection with hierarchies. A fundamental question here is the following: does the matching polytope have any compact semidefinite extended formulation?

Thomas: By a counting argument, we know that there exist matroid polytopes with no compact extended formulations, but we do not know which. Having an explicit example of such a class of matroids would be interesting. It would also be nice to have simpler proofs of results in the area, so that they can be taught in graduate classes. For instance, we now have a short, elegant proof of the result on the TSP polytope by Volker Kaibel and Stefan Weltge [8].

Sam: For the matching polytope, Gabor Braun and Sebastian Pokutta [1] gave a proof of the lower bound by Thomas, and of some extensions, using information theory, but the proof is similarly involved. On the other hand, for the permutahedron, Michel Goemans gave a tight bound on the size of an extended formulation with an elegant argument [5]: that can also be taught in class.

Another interesting line of work is that pursued by papers such as the one by Kwan, Sauermann, and Zhao [9] on the size of extended formulations of random polytopes in low dimension, for which they have pretty surprising results!

Thank you guys! We are sure the readers will appreciate your thoughts.

Interview by Yuri Faenza, Joseph Paat, Stefan Weltge.

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