Limited revision multistage stochastic programming

Chengwenjian Wang* [1] Alexander S. Estes [2] Jean-Philippe P. Richard* [1]

[1]University of Minnesota, Twin Cities, ISyE [2]University of Maryland, Robert H. Smith School of Business

Main messages

- Introduce the ***K*-revision constraint** for multistage stochastic IP that improves predictability while preserving adaptability.
- **Two IPs**: (CP) tracks active plans and is better for large scenario trees; (ST) rules out forbidden subtrees and is better for small trees.
- In **GDP**, 1-revision policies have small objective loss.

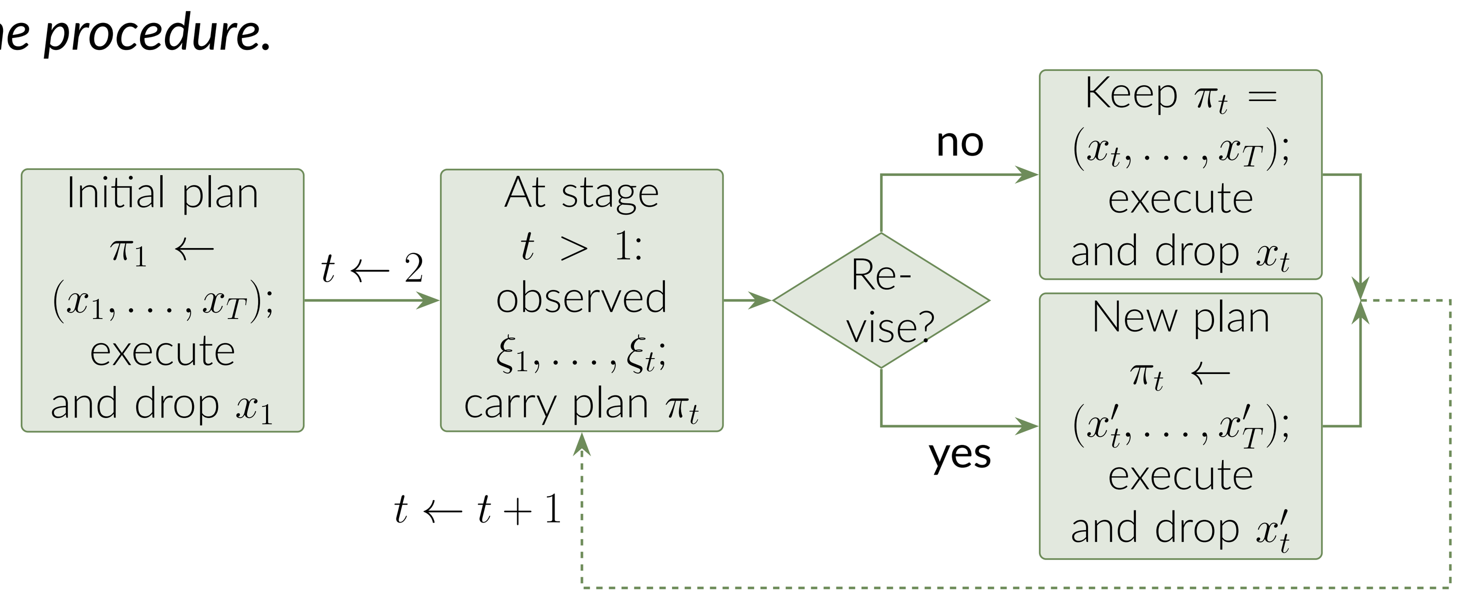
Motivation: ground delay program (GDP)

When scheduled arrivals exceed airport capacity, the FAA can hold flights on the ground to avoid costly airborne holding.

In practice, the FAA issues an ***initial GDP***, assigns delays to all relevant flights. This GDP can be revised but the FAA does so ***infrequently***, as passengers and airlines value ***predictability***.

Plan ahead and revise infrequently

The procedure.

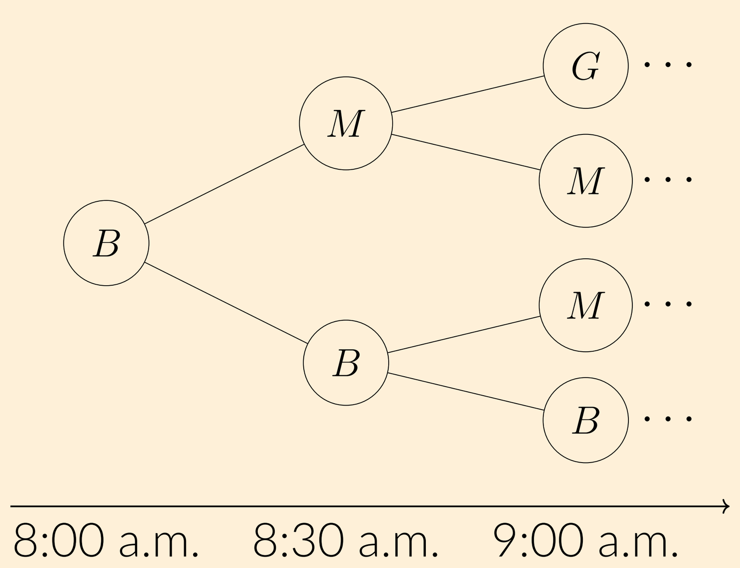


Definition. *K-revision constraint* restricts that no more than K revisions can be made for each scenario. We call such a policy x ***K-revisable***.

Important remarks.

- Assumption: uncertainty is described by a *scenario tree*.
- Goal: apply to any multistage stochastic IP on a scenario tree.
  - Strongly NP-hard, even for minimal K-revision setting.
- Modeling choice: only apply K-revision constraint on ***strategic*** decisions.

Example. In GDP, a scenario tree can be as follows



Weather condition code
B: bad
M: marginal
G: good

The strategic decision is whether a flight take off in a given period [1, 3].

Complete plan formulation

$\pi_{v,t}$: the plan active at node v for stage t where $t \geqslant \tau(v)$.
r_v : indicator that a revision is counted at node v.

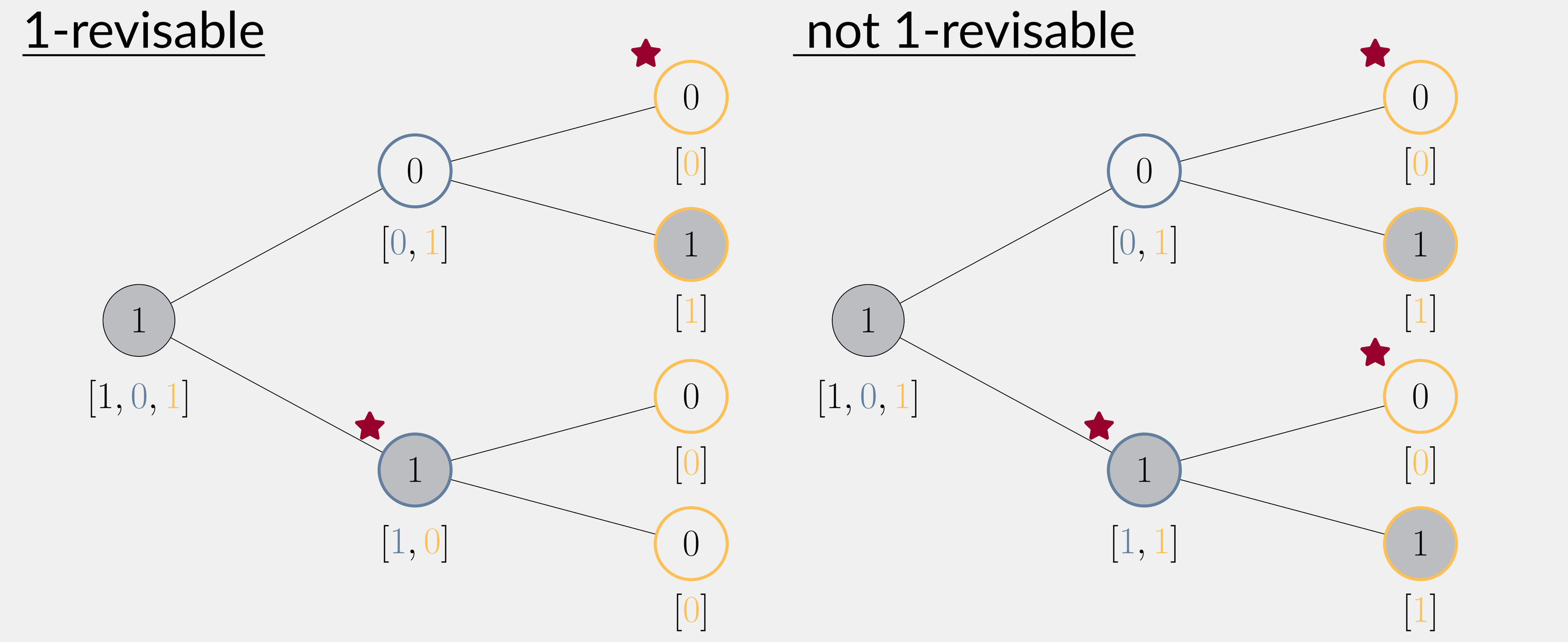
$$
\begin{aligned}
& x_v = \pi_{v,\tau(v)}, && \forall v \in \mathcal{N} \\
& r_v \geqslant \left|\pi_{v,t} - \pi_{\mathrm{pa}(v),t}\right|, && \forall v \in \mathcal{N}\setminus\{\rho\}, \forall t \in [\tau(v):T] \\
& \textstyle\sum_{v\in\omega} r_v \leqslant K, && \forall \omega \in \Omega \\
& x_v,\ \pi_{v,t},\ r_v \in \{0,1\}, && \forall v \in \mathcal{N}, \forall t \in [\tau(v):T]
\end{aligned}
\qquad \text{(CP)}
$$

Note. $\mathcal{N}$: set of nodes in the scenario tree, Ω: set of scenarios (*i.e.*, root-to-leaf paths), $\tau(v)$: the stage of node v, $\mathrm{pa}(v)$: the parent node of node v.

Properties and strengthening of (CP).

- Not sharp for a perfect binary tree with $T = 3$ and $K = 1$.
- Empirically strong for tall trees.
- Size can be reduced from $\mathcal{O}(|\mathcal{N}|T)$ to $\mathcal{O}(|\mathcal{N}|)$ $\rightarrow$ **(CP+)**.
- Can be made stronger by a new family of facet-defining inequalities $\rightarrow$ **(CP++)**.

Example: 1-revisable and not 1-revisable policies



Note. A node v is shaded gray if $x_v = 1$, whereas a node is blank if $x_v = 0$. The vectors beneath the nodes specify a plan assignment. A node is marked with a star if a revision occurs.

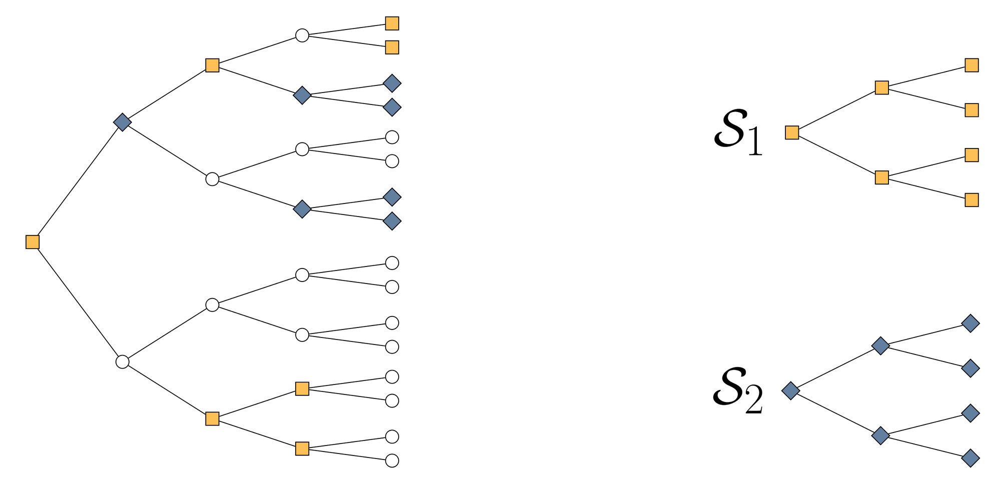
Subtree formulation

Question. Can we formulate a constraint using only the x variables?

Intuitively, to satisfy 1-revision, avoid the structure appearing in the **not** 1-revisable instance above.

Theorem. A policy x is K-revisable iff there are no height-$(K+1)$ ***ELBE subtrees*** such that each pair of siblings has different x-value.

Example. *Height-2 equi-level binary embedded (ELBE) subtrees*.



$$
\begin{aligned}
& \textstyle\sum_{\{u,v\}\in \mathrm{sib}(\mathcal{S})} \left|x_u - x_v\right| \leqslant 2^{K+1} - 2 && \forall \mathcal{S} \in \mathscr{S}_{K+1}(\mathcal{T}) \\
& x_v \in \{0,1\}, && \forall v \in \mathcal{N}
\end{aligned}
\qquad \text{(ST)}
$$

Note. $\mathscr{S}_{K+1}$: the set of ELBE subtrees with height $K+1$.

Properties and strengthening of (ST).

- Strong for short trees (ideal for $T \leqslant K+2$).
- Can be weak for tall trees.
- Large size can be mitigated through an extended formulation: **(STDP)**.

Numerical experiments

Capacity planning problems

K	T	$\lvert\mathcal{N}\rvert$	time CP+	CP++	STDP
1	2	7	**0.78**	0.89	0.88
	3	15	8.21	7.61	**6.36**
	3	20	150.4	123.80	**68.2**
	4	25	415.89	255.73	**166.59**
	4	31	1000	1000	1000
	5	36	1000	1000	1000
2	3	15	4.952	5.254	**4.801**
	3	20	**50.21**	64.32	54.74
	4	25	73.25	63.04	**59.26**
	4	31	966.97	809.56	**759.88**
	5	36	693.70	726.81	**581.29**

GDP planning problems

statistic	Value	Loss
Mean	4.17%	1.92%
Median	1.37%	0.03%
Maximum	25.5%	16.2%
$\# = 0$	2	16
$\#$ in $(0, 1\%)$	25	34
$\#$ in $[1\%, 10\%)$	28	9
$\# \geqslant 10\%$	8	4

Note. Value of 1-revision is compared with the partial adaptive model [2]. Loss of 1-revision is compared with the classic multistage stochastic programming model.

Results interpretation.

- (CP)s are better for large and tall trees.
- (ST)s are better for short and small trees.
- Adding the K-revision constraint incurs a relatively small loss in GDP.

*Acknowledgments. C. Wang and J.-P. P. Richard are supported by AFOSR grant FA9550-23-1-0451.

[1] Alexander S. Estes and Michael O. Ball. Equity and strength in stochastic integer programming models for the dynamic single airport ground-holding problem. 54(4):944–955, July 2020.

[2] Sezen Ece Kayacık, Beste Basciftci, Albert H. Schrotenboer, and Evrim Ursavas. Partially adaptive multistage stochastic programming. 321(1):192–207, February 2025.

[3] Avijit Mukherjee and Mark Hansen. A dynamic stochastic model for the single airport ground holding problem. 41(4):444–456, November 2007.