Hybrid Quantum-Classical Decomposition for Propellant-Coupled Space Vehicle Routing

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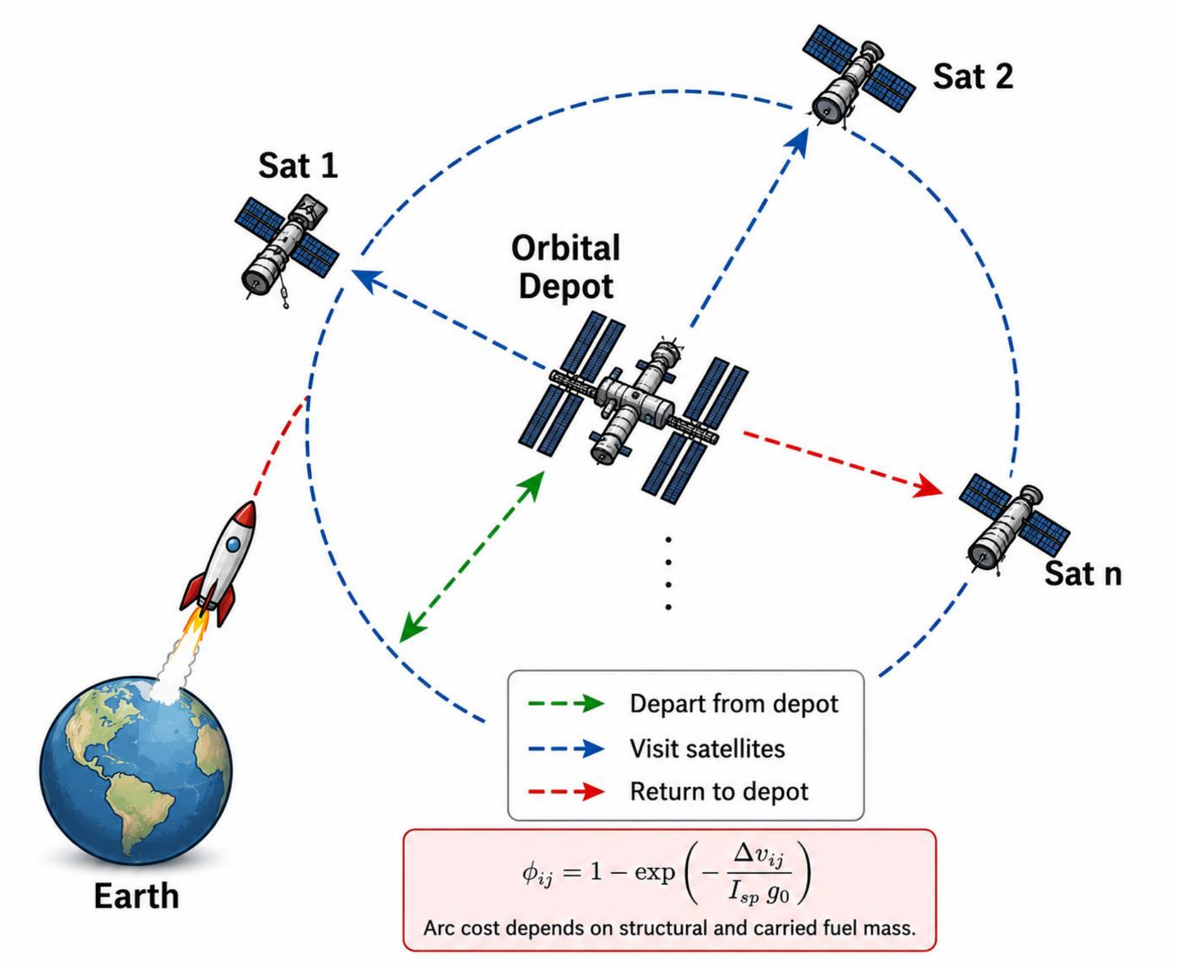
1. Motivation

- **On-orbit satellite servicing** reduces launch demand and extends satellite lifetime.
- **Propellant consumption** on each arc depends on both structural mass and carried fuel.
- **Routing decisions** influence future feasibility through the Tsiolkovsky rocket equation.
- The problem couples binary routing variables with continuous fuel-flow dynamics.



2. Background

- Reusable on orbit servicing missions require spacecraft to visit multiple satellites while satisfying fuel and operational constraints.
- Unlike classical vehicle routing problems, fuel consumption depends on orbital transfer dynamics and continuously changes system mass. Consequently, routing decisions influence future fuel requirements and route feasibility.

Key Challenge
Classical VRPs assume:
- Fixed additive travel costs

Space routing introduces:
- Mass dependent costs
- Nonlinear fuel dynamics
- Coupled routing and flow decisions

3. Problem Definition

We consider:
- One orbital depot
- Multiple target satellites
- Structural spacecraft mass
- Limited fuel capacity

Determine:
- satellite visiting sequence
- fuel allocation decisions

Objective:
Minimize total mission cost while satisfying physical feasibility.

4. Decision Variables

Routing variables: $x_{ij} \in \{0,1\}$

Fuel variables: $f_{ij} \geq 0$

where: $x_{ij} = 1$

if orbital transfer (i,j) is selected.

5. Propellant Consumption

Fuel usage follows the rocket equation:

where: $\phi_{ij} = 1 - \exp\left(-\frac{\Delta v_{ij}}{I_{sp} g_0}\right)$

Fraction of onboard mass consumed ϕ_{ij}

Velocity change requirement Δv_{ij}

Specific impulse I_{sp}

Gravitational acceleration g_0

Implication
Fuel consumption depends on route sequence and remaining mass.

6. MILP Formulation

Objective: $\min c_L(S+F) + c_O \sum_{i \neq j} \phi_{ij}(Sx_{ij} + f_{ij})$

Subject to:

Visit constraints

$$\sum_{i \neq k} x_{ik} = 1$$

$$\sum_{j \neq k} x_{kj} = 1$$

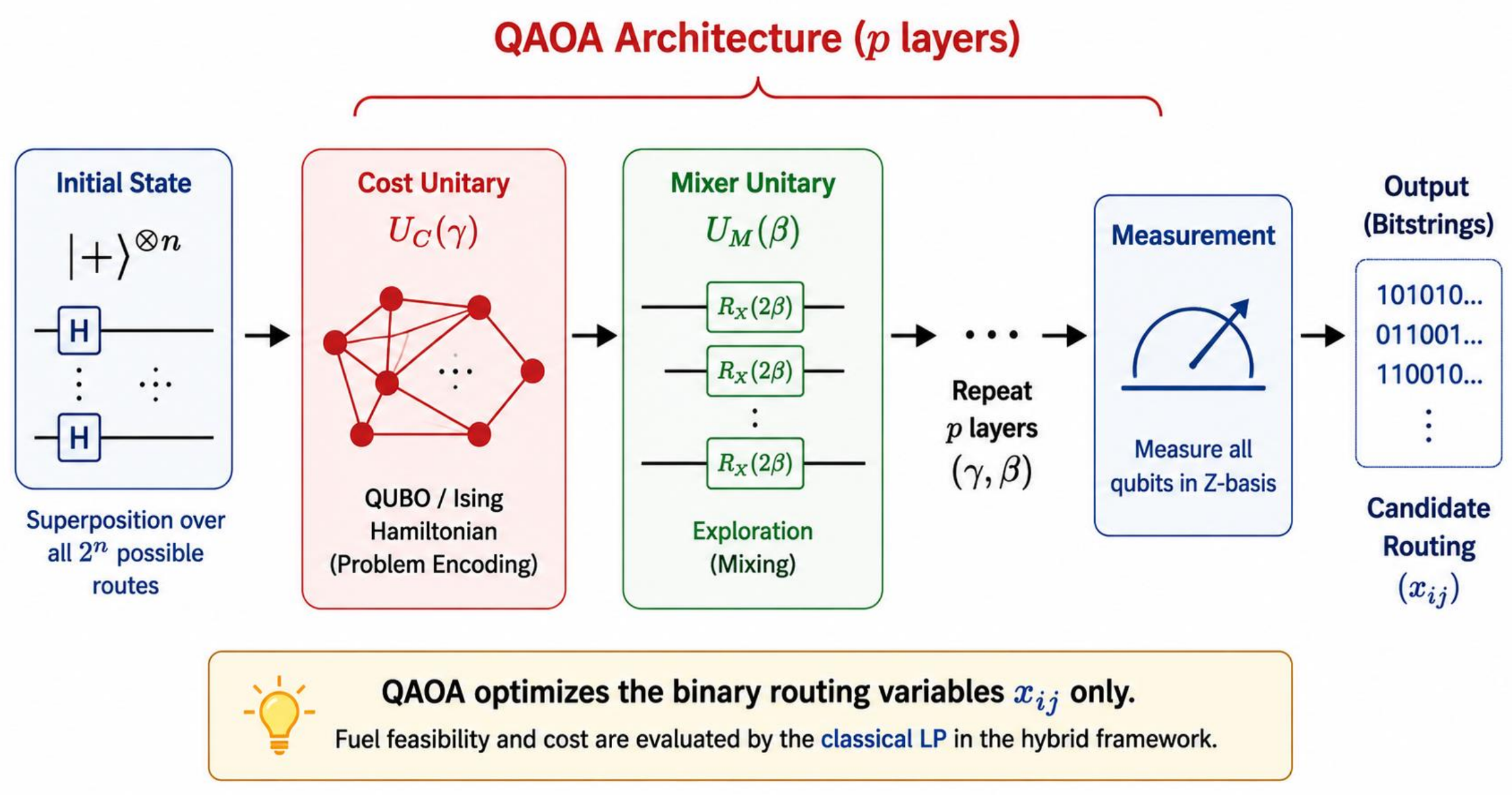
Fuel feasibility

$$(1 - \phi_{ij}) f_{ij} \geq \phi_{ij} S x_{ij}$$

Mass balance

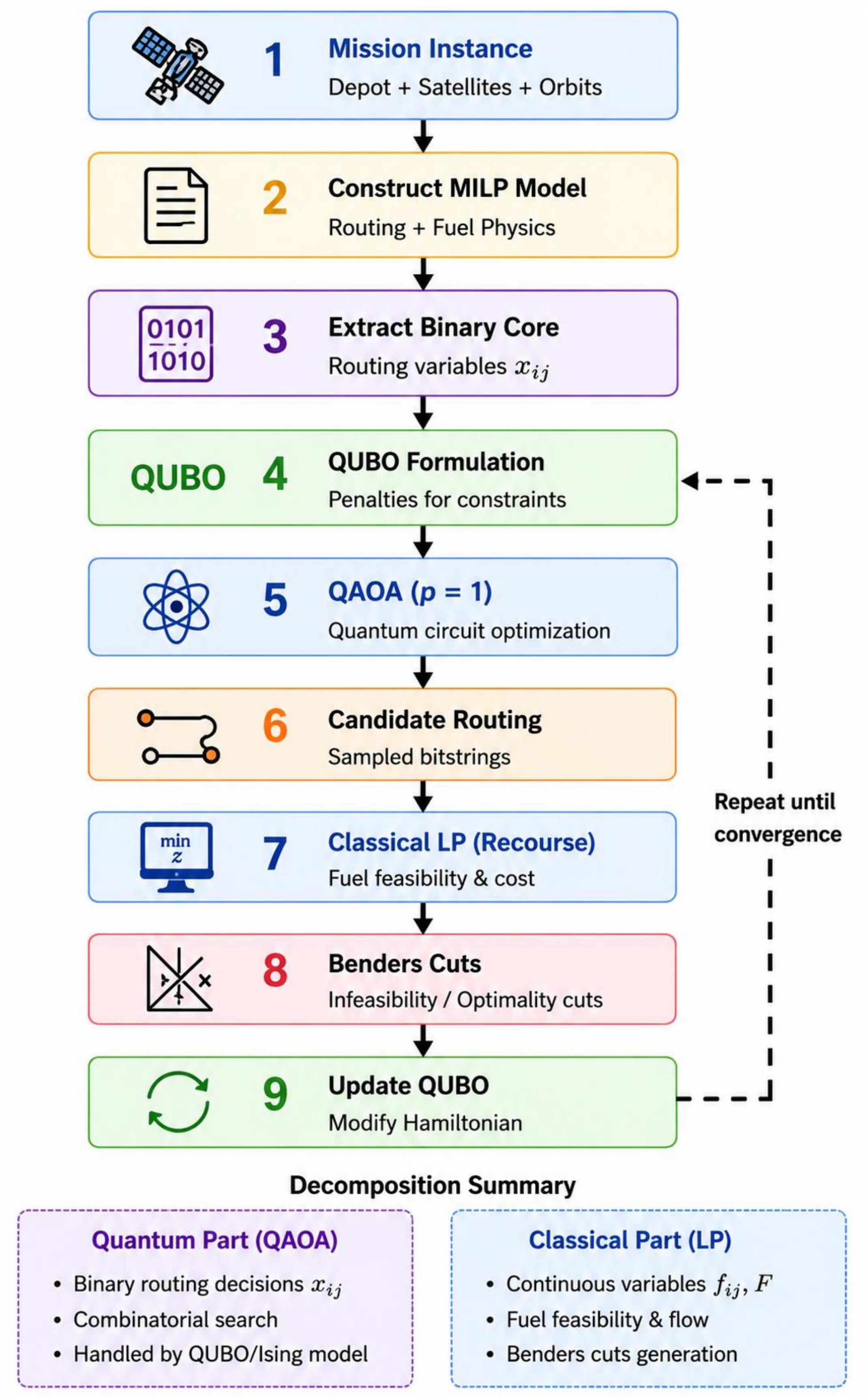
$$\sum_i (f_{ik} - \phi_{ik}(Sx_{ik} + f_{ik})) - d_k = \sum_j f_{kj}$$

7. QAOA Architecture



QAOA optimizes the binary routing variables x_{ij} only.
Fuel feasibility and cost are evaluated by the classical LP in the hybrid framework.

8. QAOA-BENDERS Framework

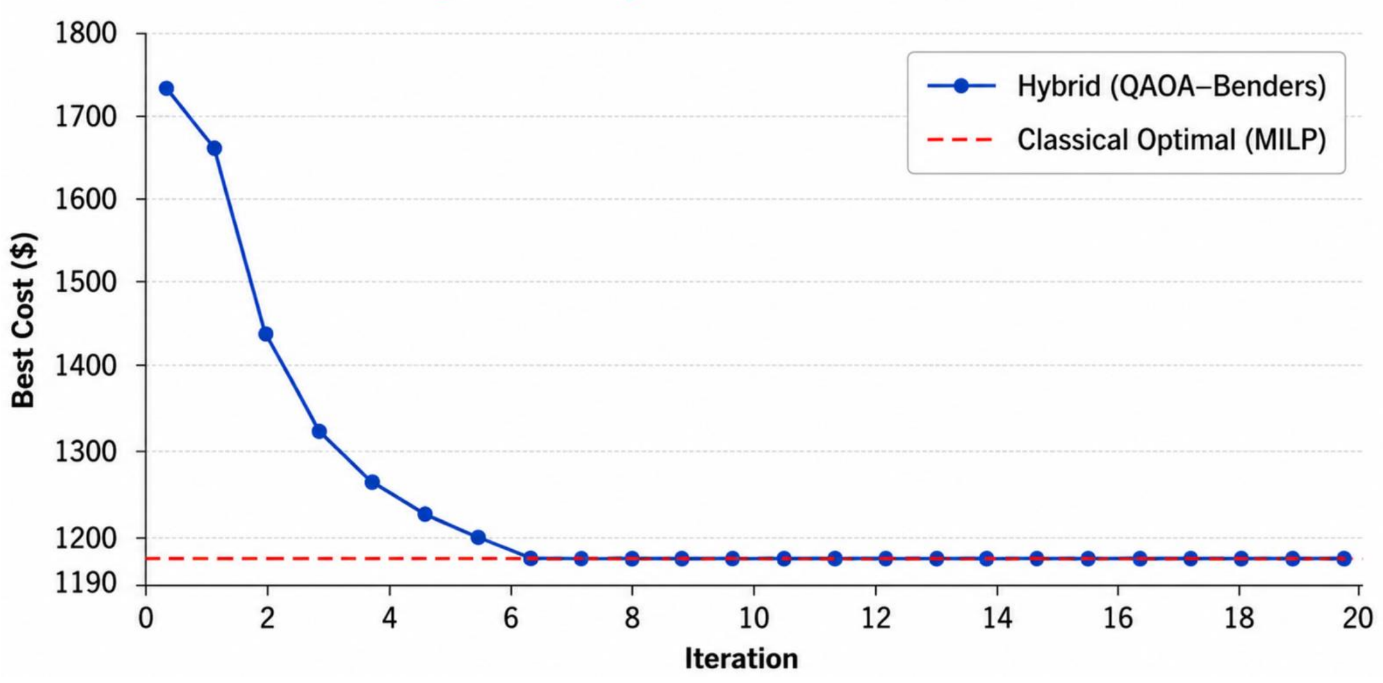


Decomposition Summary

Quantum Part (QAOA)
- Binary routing decisions x_{ij}
- Combinatorial search
- Handled by QUBO/Ising model

Classical Part (LP)
- Continuous variables f_{ij}, F
- Fuel feasibility & flow
- Benders cuts generation

9. Computational Results



	MILP	Hybrid
Optimal routing	Yes	Yes
Total cost	1191.0916	1191.0916
Fuel feasibility	Exact	Verified by LP

10. Contributions

Propellant-coupled Space VRP MILP with orbital depot, revisit.

Identification of binary routing as the sole source of integrality.

Hybrid QAOA-LP decomposition with Benders cuts feedback.

Physical feasibility guaranteed by LP recourse and
Benders cuts

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