# Contested Logistics Planning

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## 1 - Motivation

**Designing resilient logistics plans in contested environments:**

- Expeditionary and humanitarian aid logistics

## 3 - Problem Definition

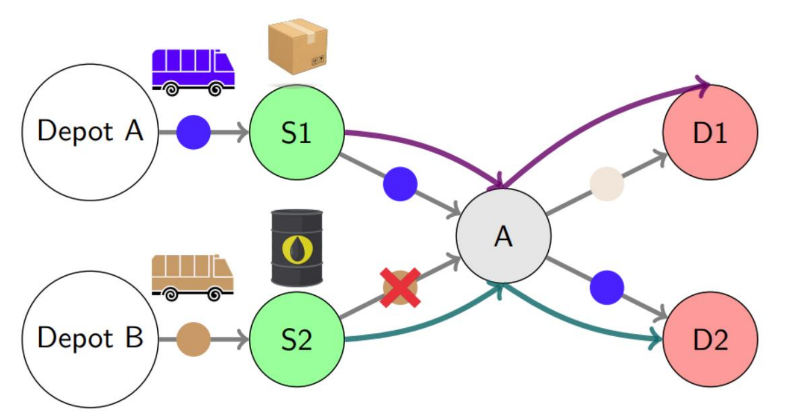
**Network Setup:** Commodities transported by vehicles on an acyclic directed graph.

**Planner's Action:** The planner defines a logistics plan.

**Attacker's Action:** The attacker observes the plan and chooses arcs to interdict.

**Impact:** Interdiction renders the vehicle and its cargo unusable for the remainder of the plan.

**No-Recourse:** The planner cannot adjust the plan after the interdiction occurs.




$$\min_{(x,f)\in\mathcal{P}} \quad \max_{I\in\mathcal{I}} \quad \text{Unmet Demand Penalty}(x,f,I)$$

**Objective:** Minimize the total penalty incurred from unmet demand in the worst-case scenario.

## 5 - Path-Based MIP

$$\min \quad z$$

$$\text{s.t.} \quad \sum_{a\in\delta^+(v)} x_{a,c} - \sum_{a\in\delta^-(v)} x_{a,c} \le s(c,v)$$

$$\sum_{\substack{\lambda\in\Lambda_k:\\ \text{start}(\lambda)=v}} f_{\lambda,k} \le \text{sup}(v,k)$$

$$\sum_{k\in K} \sum_{\substack{\lambda\in\Lambda_k:\\ \lambda\ni(a,c)}} f_{\lambda,k} \le \text{cap}(c)\cdot x_{a,c}$$

$$\sum_{a\in\delta^+(v)} x^I_{a,c} - \sum_{a\in\delta^-(v)} x^I_{a,c} \le s(c,v)$$

$$\sum_{k\in K} \sum_{\substack{\lambda\in\Lambda_k:\\ \lambda\ni(a,c)}} f^I_{\lambda,k} \le \text{cap}(c)\cdot x^I_{a,c}$$

$$\sum_{\substack{\lambda\in\Lambda_k:\\ \text{end}(\lambda)=v}} f^I_{\lambda,k} \le \text{dem}(v,k)$$

$$\sum_{c\in C} x^I_{a,c} \le 0$$

$$x^I_{a,c} \le x_{a,c}$$

$$f^I_{\lambda,k} \le f_{\lambda,k}$$

$$z \ge \sum_{v\in V}\sum_{k\in K} \text{w}(v,k)\cdot\Big(\text{dem}(v,k) - \sum_{\substack{\lambda\in\Lambda_k:\\ \text{end}(\lambda)=v}} f^I_{\lambda,k}\Big)$$

Worst-case unmet demand penalty

$$x_{a,c}\in\{0,1\}, x^I_{a,c}\ge 0, f_{\lambda,k}\ge 0, f^I_{\lambda,k}\ge 0$$

**Sets:** $V$ nodes, $A$ arcs, $C$ vehicles, $K$ commodities, $\Lambda_k$ paths for $k$
**Plan** $(x,f)\in P$: $x_{a,c}\in\{0,1\}$ vehicle routing, $f_{\lambda,k}\ge 0$ path flow
**Post-interdiction:** $x^I_{a,c}$, $f^I_{\lambda,k}\ge 0$
**Params:** $\text{cap}(c)$, $\text{sup}(v,k)$, $\text{dem}(v,k)$, $w(v,k)$

**Initial Plan Feasibility:**

- Vehicle routing must satisfy flow balance
- Commodity flow must respect initial supply and vehicle capacity limits

**Post-Interdiction Plan:**

- Delivered flow cannot exceed demand
- Interdicted vehicles cannot proceed past the interdicted arc
- No-recourse

## 7 - Best Interdiction MIP

**Most damaging interdiction plan:**

- Bilevel optimization problem
- Dualize and combine: Quadratic MIP
- Linearize using Big-M bounds

| | x, f | $x^1, f^1$ | $x^2, f^2$ | $x^3, f^3$ | ⋯ | $x^k, f^k$ | ⋯ |
|---|---|---|---|---|---|---|---|
| Initial Plan | $A_0$ | 0 | 0 | 0 | ⋯ | 0 | ⋯ |
| Interdiction 1 | $B_1$ | $C_1$ | 0 | 0 | ⋯ | 0 | ⋯ |
| Interdiction 2 | $B_2$ | 0 | $C_2$ | 0 | ⋯ | 0 | ⋯ |
| Interdiction 3 | $B_3$ | 0 | 0 | $C_3$ | ⋯ | 0 | ⋯ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋱ | ⋮ | ⋯ |
| Interdiction k | $B_k$ | 0 | 0 | 0 | ⋯ | $C_k$ | ⋯ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋯ | ⋮ | ⋱ |

$$\max_{I} \ q(I) \quad \text{s.t.} \quad \sum_{a\in A}\text{cost}(a)\cdot I_a \le B, \quad I_a\in\{0,1\}$$

$$q(\bar{I}) = \min_{x^I,f^I,z} \quad z$$

$$\text{s.t.} \quad \sum_{a\in\delta^+(v)} x^I_{a,c} - \sum_{a\in\delta^-(v)} x^I_{a,c} \le s(c,v)$$

$$x^I_{a,c} \le 1 - \bar{I}_a$$

$$x^I_{a,c} \le \bar{x}_{a,c}$$

$$f^I_{\lambda,k} \le \bar{f}_{\lambda,k}$$

$$\sum_{k\in K}\sum_{\substack{\lambda\in\bar{\Lambda}_k:\\ \lambda\ni(a,c)}} f^I_{\lambda,k} \le \text{cap}(c)\cdot x^I_{a,c}$$

$$\sum_{\substack{\lambda\in\bar{\Lambda}_k:\\ \text{end}(\lambda)=v}} f^I_{\lambda,k} \le \text{dem}(v,k)$$

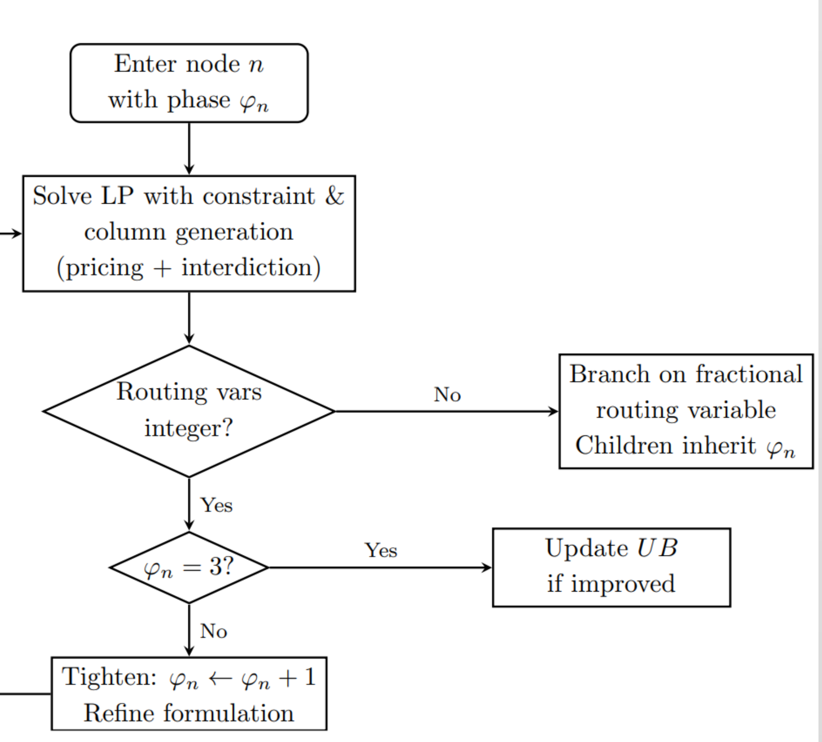
$$z \ge \sum_{v\in V}\sum_{k\in K} \text{w}(v,k)\Big(\text{dem}(v,k) - \sum_{\substack{\lambda\in\bar{\Lambda}_k:\\ \text{end}(\lambda)=v}} f^I_{\lambda,k}\Big)$$

$$x^I_{a,c}\ge 0, \quad f^I_{\lambda,k}\ge 0$$

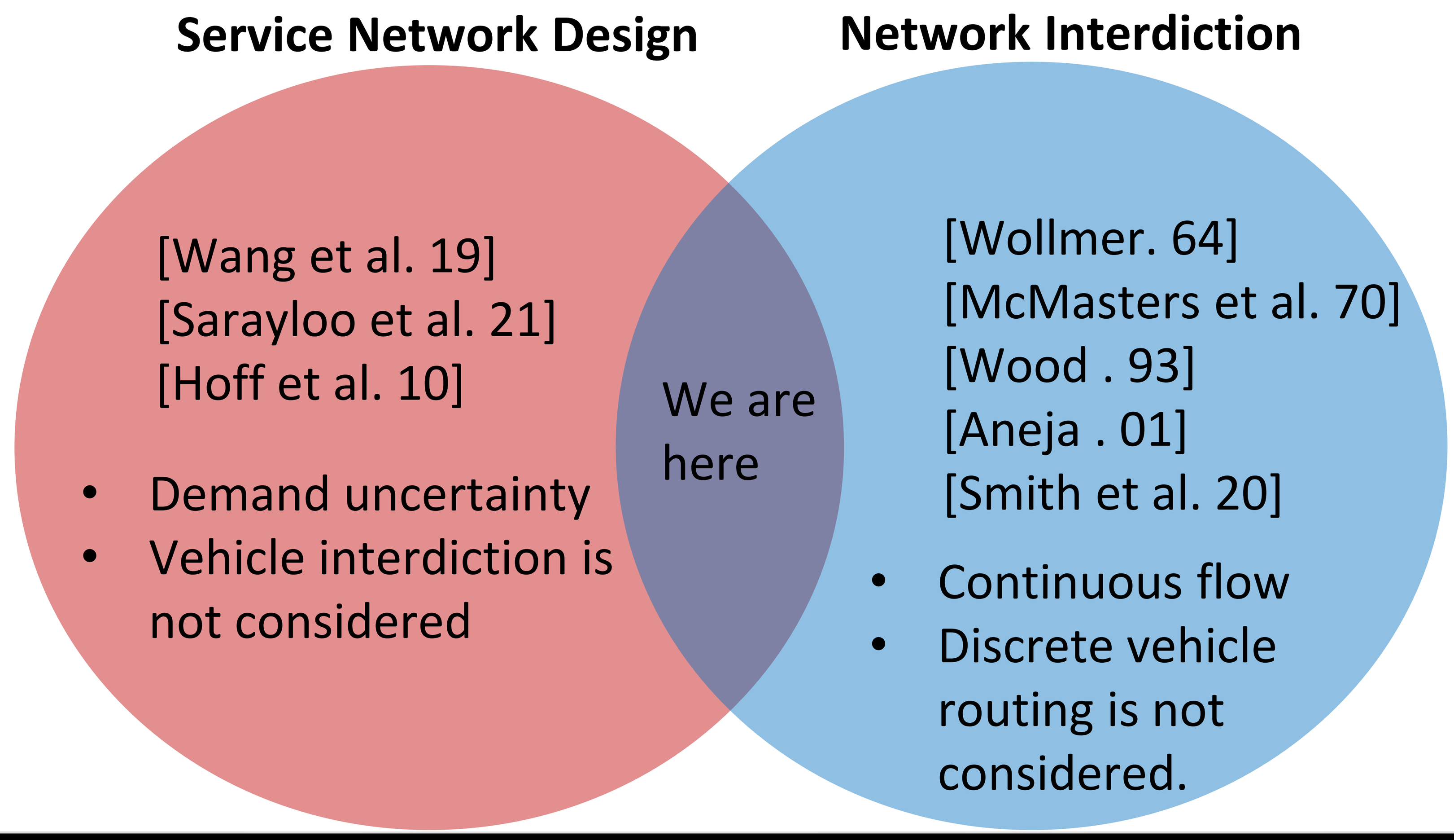
## 9 - Acceleration Techniques – Hierarchical Relaxation

**Three Phases (cheap → exact):**

- **Phase 1:** Vehicle-type arc-based LP. Aggregates vehicles by capacity. No path pricing, cheapest LP. Branch on vehicle-type routing $x_{a,l}$.
- **Phase 2:** Individual-vehicle arc-based LP. Disaggregates to per-vehicle routing $x_{a,c}$ .No pricing.
- **Phase 3:** Path-based LP (exact). Adds transportation path pricing.

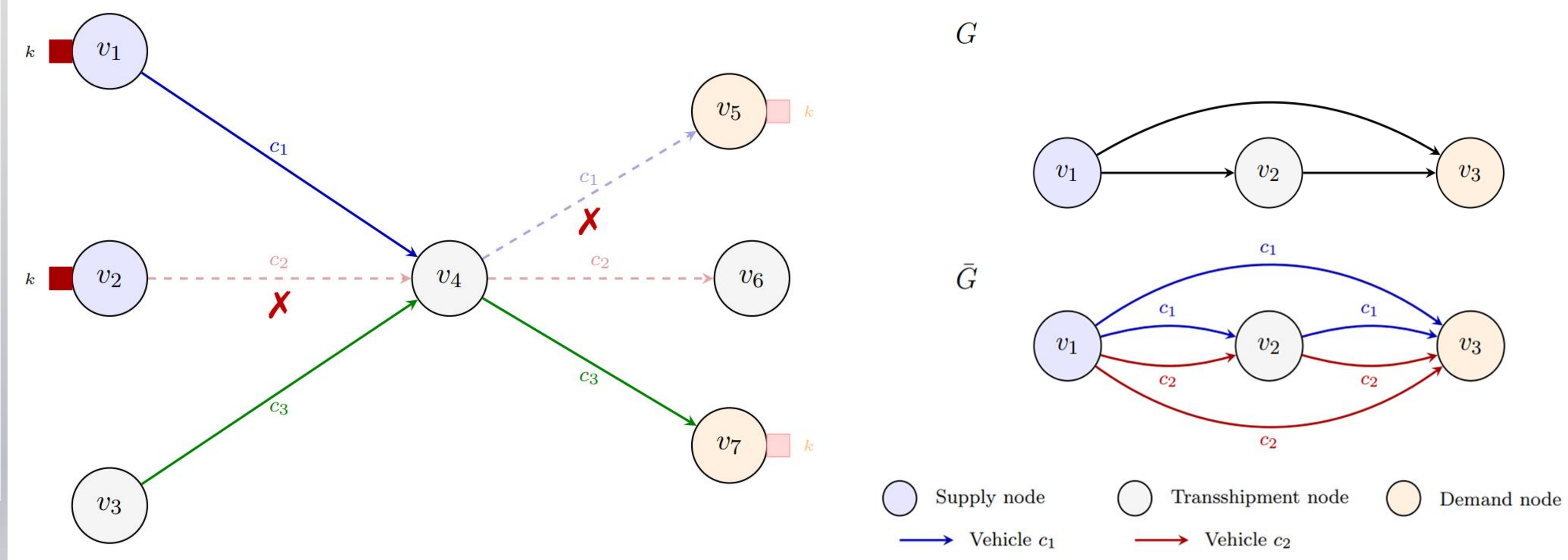



## 2 - Related Literature

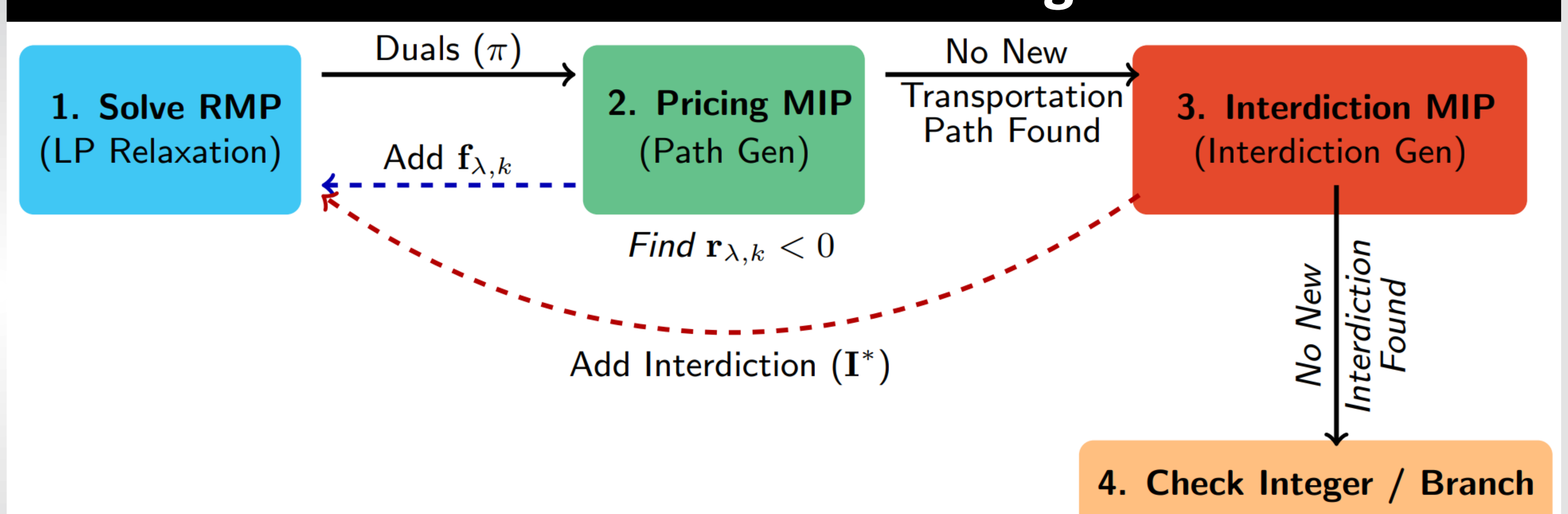



## 4 - Why a Path-Based Formulation

- Arc-based formulation cannot capture the no-recourse assumption
- Commodity flows must be indexed by transportation paths




## 6 - Branch-Price-and-Cut Algorithm



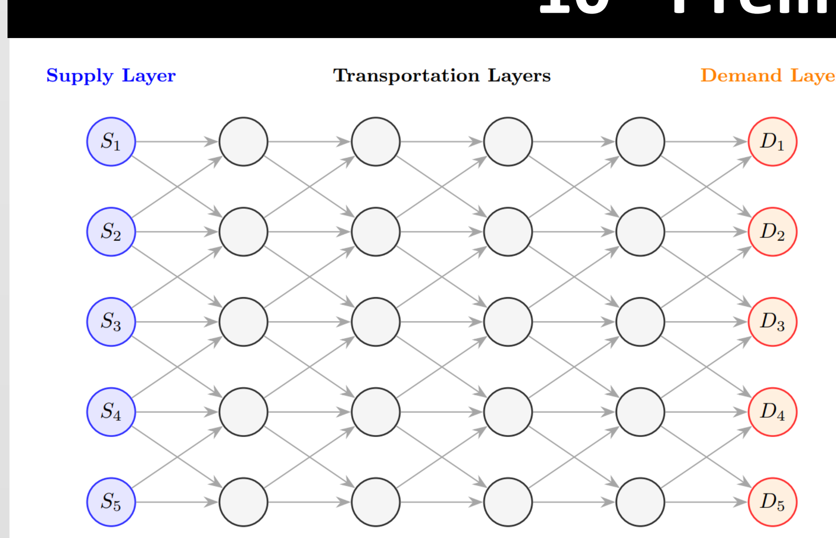

## 8 - Transportation Path Pricing

**Simultaneous constraint and column generation:**

- Formulating the pricing problem on the transportation graph.
- Construct missing dual variables in the pricing problem.

| | Known vars | | | Generated | | Not yet generated | | | | | | RHS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $x$ | $x^I$ | $z$ | $\{f_{\lambda,k}\}_{\bar{\Lambda}}$ | $\{f^I_{\lambda,k}\}_{\bar{\Lambda}}$ | $f_{\lambda^1,k}$ | $f^I_{\lambda^1,k}$ | $f_{\lambda^2,k}$ | $f^I_{\lambda^2,k}$ | ⋯ | | |
| $\text{LP}(\bar{\Lambda},\bar{\mathcal{I}},\mathcal{T})$ constraints | * | * | * | * | * | * | * | * | * | ⋯ | ≤, ≥ | * |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| No-Recourse $(\lambda^1,k)$ | 0 | 0 | 0 | 0 | 0 | −1 ⋮ −1 | $I_{\lvert\bar{\mathcal{I}}\rvert}$ | 0 | 0 | 0 | ≤ | 0 |
| No-Recourse $(\lambda^2,k)$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | −1 ⋮ −1 | $I_{\lvert\bar{\mathcal{I}}\rvert}$ | 0 | ≤ | 0 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | | | | | ⋱ | ≤ | ⋮ |
| Sign | 0 | 0 | 0 | 0 | 0 | | | $I$ | | | ≥ | 0 |

## 10 - Preliminary Results




**Experimental Setup:**

- 6x5 Layered network
- |V| = 30, |E| = 65, |C| = 10, |K| = 5
- 30 min runtime limit
- Solver: Gurobi
- UB: LP rounding heuristic

| Budget $B$ | Metric | Number of Vehicle Types $\lvert L\rvert$ | | | | |
|---|---|---|---|---|---|---|
| | | $\lvert L\rvert = 1$ | $\lvert L\rvert = 2$ | $\lvert L\rvert = 3$ | $\lvert L\rvert = 4$ | $\lvert L\rvert = 5$ |
| $B = 1$ | LB | 12,723.0 | 14,913.5 | 13,053.0 | 13,311.5 | 12,777.5 |
| | UB | 12,723.0 | 14,975.0 | 13,371.6 | 13,443.8 | 12,941.3 |
| | Gap | 0.00% | 0.41% | 2.43% | 0.99% | 1.29% |
| $B = 2$ | LB | 23,533.0 | 23,533.0 | 23,533.0 | 23,533.0 | 23,533.0 |
| | UB | 23,977.0 | 23,809.6 | 23,665.0 | 23,881.0 | 24,229.0 |
| | Gap | 1.88% | 1.17% | 0.56% | 1.48% | 2.96% |
| $B = 3$ | LB | 28,230.3 | 28,322.5 | 28,230.3 | 28,230.3 | 28,230.3 |
| | UB | 30,668.7 | 31,550.3 | 32,470.0 | 32,015.9 | 33,027.5 |
| | Gap | 8.64% | 11.40% | 15.02% | 13.41% | 16.99% |

**Future Work:**

- Benders decomposition
- Upper bound heuristic