A bilevel approach for compensation and routing decisions in last-mile delivery

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Application
Peer-to-peer logistic platforms
Peer-to-peer logistic platforms
Peer-to-peer logistic platforms
Our scenario

A set $I$ of items to be delivered to a set $V$ of customers.

A platform receives a price $p_i$ for each item to be delivered.

Given a set $K$ of potential carriers (e.g., occasional drivers), the platform searches for carriers that can deliver subsets of $I$, and pays to carrier $k$ a compensation $\bar{p}_{i,k}$ for each delivered item, i.e., for each served customer. Each carrier $k$ pays $c_{ij}^k$ to go from costumer $i$ to costumer $j$. 
Our scenario

The platform proposes to each carrier $k$ a set of items $P_k$ to serve and a compensation for each item $p_{i,k}$.

Each carrier $k$ receives the proposal, and, based on her net profit, decides on a subset of customers $Q_k \subseteq P_k$ to accept to serve.
Our scenario

The platform proposes to each carrier $k$ a set of items $P_k$ to serve and a compensation for each item $p_{ik}^k$.

Each carrier $k$ receives the proposal, and, based on her net profit, decides on a subset of customers $Q_k \subseteq P_k$ to accept to serve.
Optimization problem

How the platform maximizes its own profit if it has no direct control over the carriers?
Bilevel Programming
Bilevel programming

\[
\begin{align*}
\text{"min" } F(x, y) \\
\text{s.t. } G(x, y) \leq 0 \\
y \in \arg\min_{y' \in \mathcal{Y}} \{ f(x, y') \mid g(x, y') \leq 0 \}
\end{align*}
\]

Optimistic approach
\[
\min_{x \in \mathcal{X}} \min_{y \in S(x)} F(x, y)
\]

Pessimistic approach
\[
\min_{x \in \mathcal{X}} \max_{y \in S(x)} F(x, y)
\]
One way to reformulate the bilevel problem is considering the so-called value function of the lower-level problem:

\[
\varphi(x) = \min_{y'} \{ f(x, y') | g(x, y') \leq 0 \},
\]

obtaining the following single-level reformulation:

\[
\begin{align*}
\min_{x, y} & \quad F(x, y) \\
\text{s.t.} & \quad G(x, y) \leq 0 \\
& \quad g(x, y) \leq 0 \\
& \quad f(x, y) \leq \varphi(x) \\
& \quad x \in X, y \in Y.
\end{align*}
\]
The bilevel formulation
The bilevel framework

In our problem, the platform acts as the leader. The carriers act as the $|K|$ followers.

Parameters and sets:

- $V$ set of customers to be served (or equivalently items to be delivered)
- $K$ set of independent carriers
- $p_i$ price the customer $i$ pays to the platform
- $c_{ij}^k$ routing cost that carrier $k$ pays to go from customer $i$ to customer $j$
- $b^k$ maximum number of customers carrier $k$ can serve

Decisions:

- The platform proposes to each carrier $k$ a set of items $P_k$ to serve and a compensation for each item $\tilde{p}^k_i$.
- Each carrier $k$ receives the proposal, and, based on her net profit, decides on a subset of customers $Q_k \subseteq P_k$ to accept to serve.
The bilevel framework

The goal of each carrier $k$ is maximizing her net profit:

$$\sum_{i \in Q_k} \bar{p}_i^k - \sum_{i \in Q_k} \sum_{j \in Q_k} c_{ij}^k.$$ 

The goal of the leader is making a call to the carriers, so as to maximize his profit, which is defined as:

$$\sum_{k \in K} \sum_{i \in Q_k} (p_i - \bar{p}_i^k).$$
Do we really need bilevel optimization?

- \( V = \{1, 2, 3, 4, 5, 6\} \)
- \( K = \{a, b\} \)
- \( b^k \) is 2 for all \( k \)

For each \( i \):
\((p_i, \bar{p}^i_k)\) for all \( k \)
Do we really need bilevel optimization?

In a single-level setting, the leader assigns parcels so as to maximize the profit and such that each carrier has a non-negative net profit.

The platform assigns items:
- 1 and 2 to carrier A
- 3 and 4 to carrier B

Profit = 25.5

In the bilevel setting:
- carrier A accepts only 1
- carrier B accepts both 3 and 4

Profit = 16

OVERESTIMATION
Do we really need bilevel optimization?

In another single-level setting, we discard the role of the platform and we only maximize the profit of the total set of carriers.

- carrier A serves items 4, and 5
- carrier B serves items 1 and 6

Total profit of the carriers’ alliance = 19.1

In the bilevel setting:
- carrier A accepts 4 and 5
- carrier B accepts 1 and 6
Profit = 8.9

In the optimal solution, the leader would assign items 3, 4, 1 and 6.
Profit = 16.5

UNDERESTIMATION
The bilevel framework: compensation decisions

The profit of the leader is:

\[ \sum_{k \in K} \sum_{i \in Q_k} (p_i - \overline{p}_k^i). \]

We assume that the leader cannot decide on \( \overline{p}_i^k \) directly, but has \(|M|\) different possible categories of profit margins that can choose to gain for each item.

The profit margin \( m = \frac{(p_i - \overline{p}_i^k)}{p_i} \).

Thus, for an item \( i \) and a vehicle \( k \), the net profit \( (p_i - \overline{p}_i^k) \) is, for a margin \( m \),

\[ p_{mi} := m \cdot p_i \]

Thus, for an item \( i \) and a carrier \( k \), the compensation \( \overline{p}_i^k \) is, for a margin \( m \),

\[ \overline{p}_{mi} = p_i - p_{mi} \]
Variables

**Upper-level** variables for all $i \in V$:

$X^k_{mi}$ 1 iff the margin selected by the leader for item $i$ and carrier $k$ is $m$

**Lower-level** variables for all $i \in V$:

$y^k_i$ 1 iff carrier $k$ *accepts* to serve item $i$, i.e. if $i \in Q_k$

$z^k_{ij}$ 1 iff arc $(i, j)$ is traversed by carrier $k$ to deliver items $i$ and $j$ from $Q_k$
Objective functions

Being \( m = \frac{(p_i - \bar{p}_i^k)}{p_i} \) and \( p_{mi} = mp_i = p_i - \bar{p}_i^k \), and \( \bar{p}_{mi} = p_i - p_{mi} \):

- The goal of the leader is making a call to the carriers, so as to maximize his net profit:

  \[
  \sum_{k \in K} \sum_{i \in Q_k} (p_i - \bar{p}_i^k) = \sum_{k \in K} \sum_{m \in M} \sum_{i \in V} p_{mi} X_{mi}^k y_i^k.
  \]

- The goal of each follower \( k \) is maximizing her net profit:

  \[
  \sum_{i \in Q_k} \bar{p}_i^k - \sum_{i \in Q_k} \sum_{j \in Q_k} c_{ij}^k = \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} X_{mi}^k y_i^k - \sum_{(i,j) \in A} c_{ij}^k z_{ij}^k.
  \]
The bilevel formulation

\[
\begin{align*}
\max_{X,y} & \quad \sum_{k \in K} \sum_{i \in V} \sum_{m \in M} p_{mi} x^k_{mi} y^k_i \\
\text{s.t.} & \quad \sum_{k \in K} \sum_{m \in M} x^k_{mi} \leq 1 & \forall i \in V \setminus \{0\} \\
& \quad y^k \in S^k_\Phi(X^k) & \forall k \in K \\
& \quad y^k, x^k_m \in \{0, 1\}^n & \forall m \in M, k \in K
\end{align*}
\]

where \( S^k_\Phi(X^k) \) is the set of optimal solutions of the k-th follower problem.
Follower’s problem (the Profitable Tour Problem)

The carrier (follower) \( k \) problem for a given \( \tilde{X}^k \) is formulated as:

\[
\Phi^k(\tilde{X}^k) = \max_{y,z} \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} \tilde{X}^k_{mi} y_i^k - \sum_{(i,j) \in A} c_{ij}^k z_{ij}^k \\
\text{s.t.} \quad y_i^k \leq \sum_{m \in M} \tilde{X}^k_{mi} \quad \forall i \in V \\
\sum_{i \in V} y_i^k \leq b^k \\
(y^k, z^k) \text{ is a route} \\
y^k \in \{0, 1\}^{n+1}, z^k \in \{0, 1\}^{|A|}
\]
We assume that we are in an optimistic setting, i.e., for a given choice of $\tilde{X}^k$, if follower $k$ has multiple optimal responses determined by different sets $Q_k$ of items to be delivered, she will accept to deliver the items which are more favorable to the leader:

$$\tilde{Q}_k = \arg \max_{Q_k} \left\{ \sum_{i \in Q_k} p_{mi} \tilde{X}_{mi}^k : Q_k \in S^k_{\Phi}(\tilde{X}^k) \right\}.$$
Dealing with bilinear terms

We can linearize this bilevel MINLP using the McCormick’s inequalities. We introduce additional binary variables $w^k_{mi}$ defined as $X^k_{mi}y^k_i$ and adjoin the following inequalities to the upper-level model:

\[
X^k_{mi} + y^k_i \leq w^k_{mi} + 1 \quad \forall \; m \in M, \; i \in V, \; k \in K
\]
\[
w^k_{mi} \leq X^k_{mi} \quad \forall \; m \in M, \; i \in V, \; k \in K
\]
\[
w^k_{mi} \leq y^k_i \quad \forall \; m \in M, \; i \in V, \; k \in K
\]
Linearized bilevel model

\[
\max_{X,w,y} \quad \sum_{k \in K} \sum_{i \in V} \sum_{m \in M} p_{mi} w_{mi}^k
\]

\[
\text{s.t.} \quad \sum_{k \in K} \sum_{m \in M} X_{mi}^k \leq 1 \quad \forall i \in V \setminus \{0\}
\]

\[
y^k \in S_{\Phi}^k(X^k) \quad \forall k \in K
\]

McCormick Inequalities

\[
X_{m}, w_{m}^k \in \{0, 1\}^n \quad \forall m \in M, k \in K
\]

\[
y^k \in \{0, 1\}^{n+1} \quad \forall k \in K
\]
Single-level reformulations
Value function reformulation

\[
\begin{align*}
\max_{X,w,y,z} & \quad \sum_{k \in K} \sum_{i \in V} \sum_{m \in M} p_{mi} w^k_{mi} \\
\text{s.t.} & \quad \sum_{k \in K} \sum_{m \in M} X^k_{mi} \leq 1 \quad \forall i \in V \setminus \{0\} \\
& \quad \sum_{i \in V} y^k_i \leq b^k \quad \forall k \in K \\
& \quad \text{McCormick ineq.} \quad \forall m \in M, i \in V, k \in K \\
& \quad y^k_i \leq \sum_{m \in M} X^k_{mi} \quad \forall i \in V, k \in K \\
& \quad (y^k, z^k) \text{ is a route} \quad \forall k \in K \\
& \quad \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} w^k_{mi} - \sum_{(i,j) \in A} c^k_{ij} z^k_{ij} \geq \Phi^k(X^k) \quad \forall k \in K
\end{align*}
\]
Value function reformulation

\[
\begin{align*}
\max_{X, w, y, z} & \quad \sum_{k \in K} \sum_{i \in V} \sum_{m \in M} p_{mi} w_{mi}^k \\
\text{s.t.} & \quad \sum_{k \in K} \sum_{m \in M} X_{ki}^k \leq 1 \quad \forall i \in V \setminus \{0\} \\
& \quad \sum_{i \in V} y_{ki}^k \leq b_k^k \quad \forall k \in K \\
& \quad y_{ki}^k \leq \sum_{m \in M} X_{mi}^k \quad \forall i \in V, k \in K \\
& \quad (y_k^k, z_k^k) \text{ is a route} \quad \forall k \in K \\
& \quad \sum_{i \in V} \sum_{m \in M} p_{mi} w_{mi}^k - \sum_{(i,j) \in A} c_{ij}^k z_{ij}^k \geq \Phi_k^k(X_k^k) \quad \forall k \in K
\end{align*}
\]
Value function reformulation

\[
\max_{X,w,y,z} \sum_{k \in K} \sum_{i \in V} \sum_{m \in M} p_{mi} w_{mi}^k
\]

s.t.

\[
\sum_{k \in K} \sum_{m \in M} X^k_{mi} \leq 1 \quad \forall i \in V \setminus \{0\}
\]

\[
\sum_{i \in V} \sum_{m \in M} X^k_{mi} \leq b^k \quad \forall k \in K
\]

McCormick ineq.

\[
y^k_i \leq \sum_{m \in M} X^k_{mi} \quad \forall i \in V, k \in K
\]

\[(y^k, z^k) \text{ is a route} \quad \forall k \in K\]

\[
\sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} w_{mi}^k - \sum_{(i,j) \in A} c_{ij}^k z_{ij}^k \geq \Phi^k(X^k) \quad \forall k \in K
\]
Proposition

\[ \Phi^k(\tilde{X}^k) = \max_{y,z} \sum_{i \in V} \sum_{m \in M} \tilde{p}_{mi} \tilde{X}_{mi}^k y_i^k - \sum_{(i,j) \in A} c_{ij}^k z_{ij}^k \]

s.t. \[ y_i^k \leq \sum_{m \in M} \tilde{X}_{mi}^k \]
\[ \sum_{i \in V} y_i^k \leq b^k \]

\( (y^k, z^k) \) is a route

\( y^k \in \{0, 1\}^{n+1}, z^k \in \{0, 1\}^{|A|} \)
Proposition

There always exists an optimal solution of the following problem, which is also optimal for $\Phi_k(\tilde{X}^k)$:

$$
\Phi_k(\tilde{X}^k) = \max_{y,z} \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} \tilde{X}_{mi}^k y_i^k - \sum_{(i,j) \in A} c_{ij}^k z_{ij}^k
$$

s.t. $(y^k, z^k)$ is a route

$y^k \in \{0, 1\}^{n+1}$, $z^k \in \{0, 1\}^{|A|}$
Single-level reformulation

Let $P^k_{ext}$ denote the set of all the extreme points $(y^k, z^k)$ of the convex hull of the profitable tour feasible solutions determined by constraints “$(y^k, z^k)$ is a route”. It holds:

$$
\Phi^k(X^k) = \max_{(\hat{y}^k, \hat{z}^k) \in P^k_{ext}} \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} x^k_{mi} \hat{y}^k_i - \sum_{(i,j) \in A} c_{ij}^k \hat{z}^k_{ij}
$$

Thus, by replacing value function constraint for each $k$

$$
\sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} w^k_{mi} - \sum_{(i,j) \in A} c_{ij}^k \hat{z}^k_{ij} \geq \Phi^k(X^k)
$$
Single-level reformulation

Let $P_{ext}^k$ denote the set of all the extreme points $(y^k, z^k)$ of the convex hull of the profitable tour feasible solutions determined by constraints “$(y^k, z^k)$ is a route.” It holds:

$$
\Phi^k(X^k) = \max_{(\hat{y}^k, \hat{z}^k) \in P_{ext}^k} \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} X^k_{mi} \hat{y}^k_i - \sum_{(i,j) \in A} c_{ij}^k \hat{z}^k_{ij}
$$

Thus, by replacing value function constraint for each $k$

$$
(*) \sum_{i \in V} \sum_{m \in M} \tilde{p}_{mi} w^k_{mi} - \sum_{(i,j) \in A} c_{ij}^k z^k_{ij} \geq \sum_{i \in V} \sum_{m \in M} \tilde{p}_{mi} X^k_{mi} \hat{y}^k_i - \sum_{(i,j) \in A} c_{ij}^k \hat{z}^k_{ij}, \quad \forall (\hat{y}^k, \hat{z}^k) \in P_{ext}^k
$$

we obtain a single-level reformulation of our problem.
Single-level reformulation

Let $P_{ext}^k$ denote the set of all the extreme points $(y^k, z^k)$ of the convex hull of the profitable tour feasible solutions determined by constraints “$(y^k, z^k)$ is a route”. It holds:

$$\Phi^k(X^k) = \max_{(\hat{y}^k, \hat{z}^k) \in P_{ext}^k} \sum_{i \in V} \sum_{m \in M} p_{mi}X^k_{mi}\hat{y}^k_i - \sum_{(i,j) \in A} c_{ij}^k \hat{z}^k_{ij}$$

Thus, by replacing value function constraint for each $k$

$$\sum_{i \in V} \sum_{m \in M} p_{mi}w^k_{mi} - \sum_{(i,j) \in A} c_{ij}^k \hat{z}^k_{ij} \geq \sum_{i \in V} \sum_{m \in M} p_{mi}X^k_{mi}\hat{y}^k_i - \sum_{(i,j) \in A} c_{ij}^k \hat{z}^k_{ij}, \quad \forall (\hat{y}^k, \hat{z}^k) \in P_{ext}^k$$

we obtain a single-level reformulation of our problem.

Exponentially many!
Separation procedure

- Relax constraints (*) from the bilevel problem, finding solution \((\tilde{X}^k, \tilde{w}^k, \tilde{y}^k, \tilde{z}^k)\)
- Solve the profitable tour problem (lower level) for \(X^k = \tilde{X}^k\) for each \(k\), obtaining solution \((\hat{y}^k, \hat{z}^k)\) with optimal value \(\hat{\Phi}^k\)
- If it exists a \(k\) such that
  \[
  \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} \hat{w}_{mi}^k - \sum_{(i,j) \in A} c_{ij}^k \hat{z}_{ij}^k < \hat{\Phi}^k,
  \]
add the cut
  \[
  \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} w_{mi}^k - \sum_{(i,j) \in A} c_{ij}^k z_{ij}^k \geq \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} X_{mi}^k \hat{y}_{i}^k - \sum_{(i,j) \in A} c_{ij}^k z_{ij}^k
  \]
to the master problem. Otherwise, the obtained solution is optimal for the original bilevel formulation.
Projection of $z$ variable
Projecting out the z variable

Since the platform profit is not depending on the route followed by each carrier, there is no need to consider $z$ variables at the master level:

$$\max_{X,w,y,z} \sum_{k \in K} \sum_{i \in V} \sum_{m \in M} p_{mi} w_{mi}^k$$

s.t. $$\sum_{k \in K} \sum_{m \in M} X_{ki} \leq 1 \quad \forall i \in V \setminus \{0\}$$

$$\sum_{i \in V} y_{ki} \leq b_k \quad \forall k \in K$$

McCormick ineq. $$\forall m \in M, i \in V, k \in K$$

$$y_{ki}^k \leq \sum_{m \in M} X_{mi}^k \quad \forall i \in V, k \in K$$

$$(y_k^k, z_k^k)$$ is a route $$\forall k \in K$$

$$\sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} w_{mi}^k - \sum_{(i,j) \in A} c_{ij}^k z_{ij}^k \geq \Phi^k(X_k) \quad \forall k \in K$$
Projecting out the z variable

Since the platform profit is not depending on the route followed by each carrier, there is no need to consider \( z \) variables at the master level:

\[
\begin{align*}
\max_{X,w,y,\theta} & \quad \sum_{k \in K} \sum_{i \in V} \sum_{m \in M} p_{mi} w_{mi}^k \\
\text{s.t.} & \quad \sum_{k \in K} \sum_{m \in M} X_{mi}^k \leq 1 \quad \forall i \in V \setminus \{0\} \\
& \quad \sum_{i \in V} y_{ki}^k \leq b^k \quad \forall k \in K \\
& \quad \text{McCormick ineq.} \quad \forall m \in M, i \in V, k \in K \\
& \quad y_{ki}^k \leq \sum_{m \in M} X_{mi}^k \quad \forall i \in V, k \in K \\
& \quad \theta^k \geq c_{TSP}(y^k) \quad \forall k \in K \\
& \quad \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} w_{mi}^k - \theta^k \geq \Phi^k(X^k) \quad \forall k \in K
\end{align*}
\]
Projecting out the z variable

Since the platform profit is not depending on the route followed by each carrier, there is no need to consider $z$ variables at the master level:

$$
\max_{X, w, y, \theta} \sum_{k \in K} \sum_{i \in V} \sum_{m \in M} p_{mi} w^k_{mi} \\
\text{s.t.} \sum_{k \in K} \sum_{m \in M} X^k_{mi} \leq 1 \quad \forall i \in V \setminus \{0\} \\
\sum_{i \in V} y^k_i \leq b^k \quad \forall k \in K \\
\text{McCormick ineq.} \quad \forall m \in M, i \in V, k \in K \\
y^k_i \leq \sum_{m \in M} X^k_{mi} \quad \forall i \in V, k \in K \\
\theta^k \geq c_{TSP}(y^k) \quad \forall k \in K \\
\sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} w^k_{mi} - \theta^k \geq \Phi^k(X^k) \quad \forall k \in K 
$$

Exponentially many!
Heuristic warm-start procedure
Heuristic algorithm

o We solve the problem without margin decision, setting the compensation to $ar{p}_i^k = (1 - m_{min}) p_i$ for all $i$ and $k$.

o We obtain the optimal solution in terms of assignment of the leader $\hat{x}^k$, and acceptance and routing decisions of the followers $\hat{T}^k$.

o We solve the “only-pricing problem”:

$$
\max_X \sum_{k \in K} \sum_{m \in M} \sum_{i \in V(\hat{T}^k)} p_{mi} X_{mi}^k \\
\text{s.t.} \sum_{m \in M} X_{mi}^k = \hat{x}_i^k \quad \forall i \in V, k \in K \\
\sum_{m \in M} \sum_{i \in V(\hat{T}^k)} p_{mi} X_{mi}^k \leq \sum_{i \in V(\hat{T}^k)} p_i - C(\hat{T}^k) \quad \forall k \in K,
$$

o We obtain the optimal solution in terms of margin decisions $\hat{X}$.

o We solve the problem without margin decision, setting the compensation to $ar{p}_i^k = p_i - \sum_{m \in M} \hat{x}_{mi}^k p_{mi}$ for all $i$ and $k$. 
Computational results
Instances

- **Chao**’s instances for the OP: number of customers ranging from 21 to 66
- **Solomon** instances for the VRPTW: number of customers ranging from 20 to 35
  - 2, 3 or 4 carriers
  - Different margins sets: {0.2,0.5}; {0.5,0.9}; {0.2,0.5,0.8}; {0.5,0.7,0.9} * these are the margins to the platform
  - Time limit 1 hour
  - CPLEX 22.1.0.0
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<th>Heuristic</th>
<th>Model (BPMD)</th>
<th>Model (BPMD-z)</th>
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## Solutions structure

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</table>
Conclusions

- The problem becomes harder to solve when margins are such that mix of them is made in the optimal solution.
- Solomon’s instances are harder – probably related to the geography of customers.
- Projecting out z variables pays off on Chao’s instances, but not on Solomon’s ones.
Thank you! 😊

Any questions?

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