A bilevel approach for compensation and routing decisions in last-mile delivery

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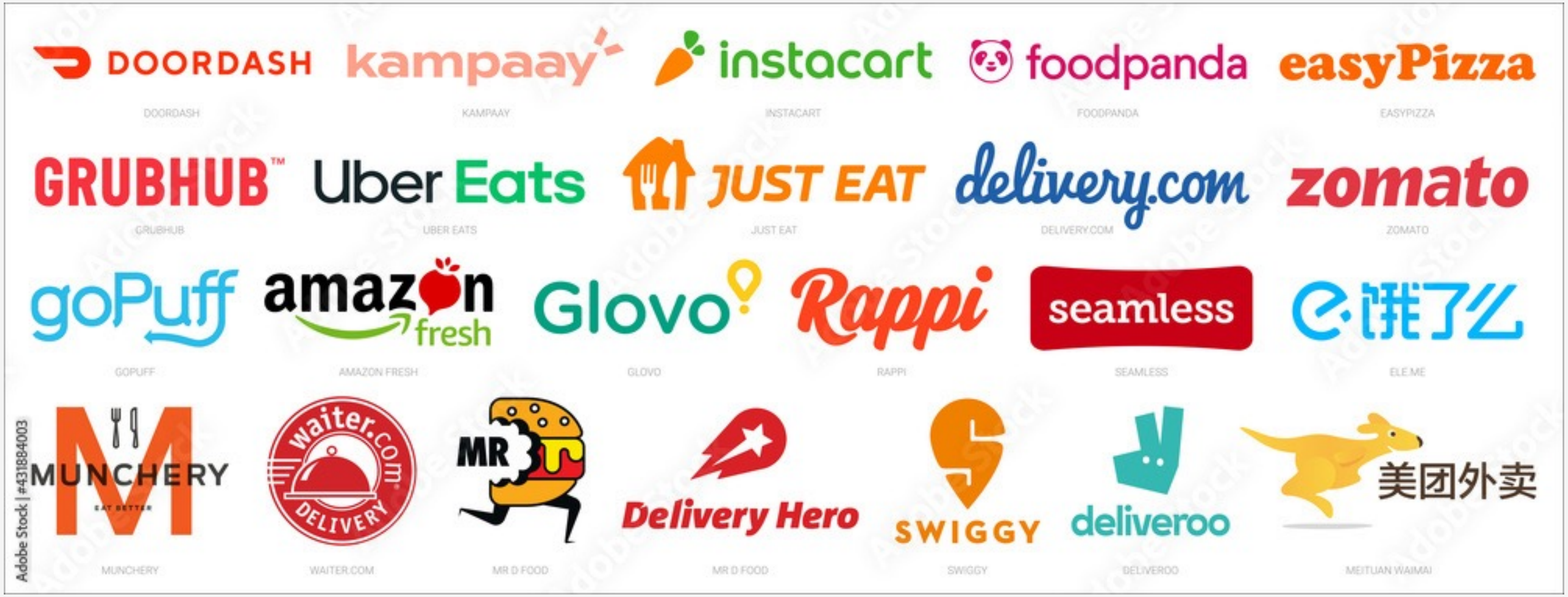
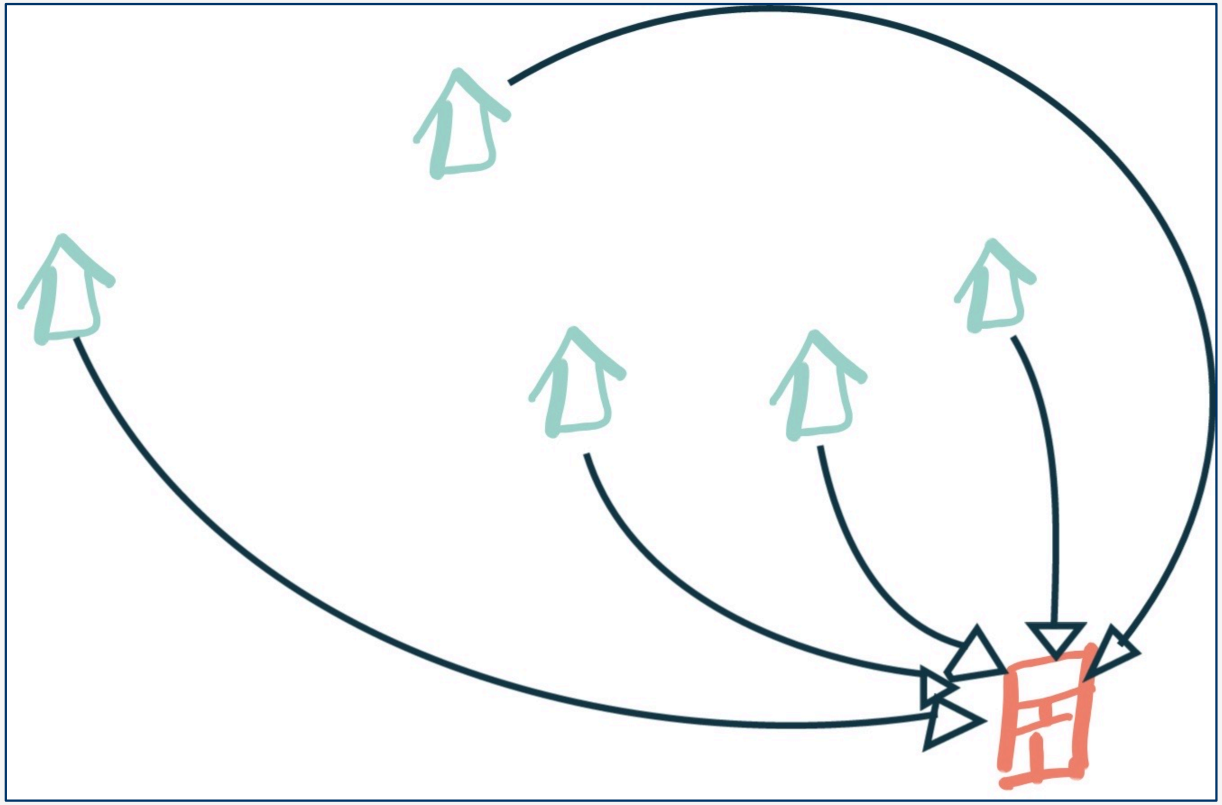
Application

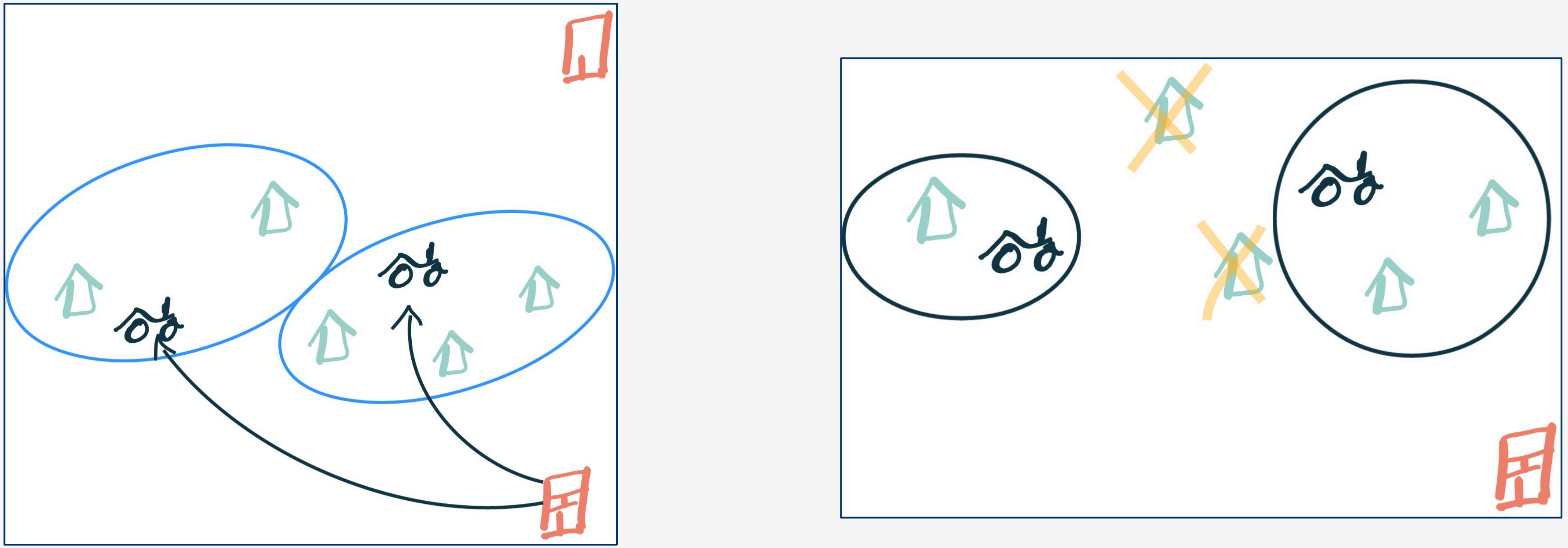
Peer-to-peer logistic platforms



Peer-to-peer logistic platforms

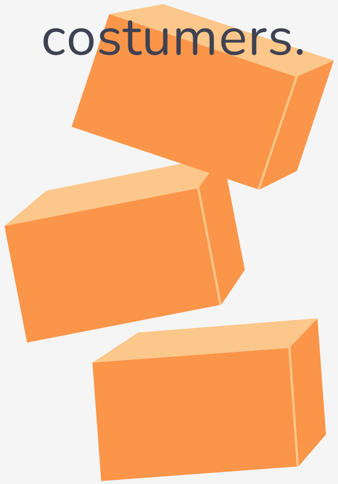


Peer-to-peer logistic platforms



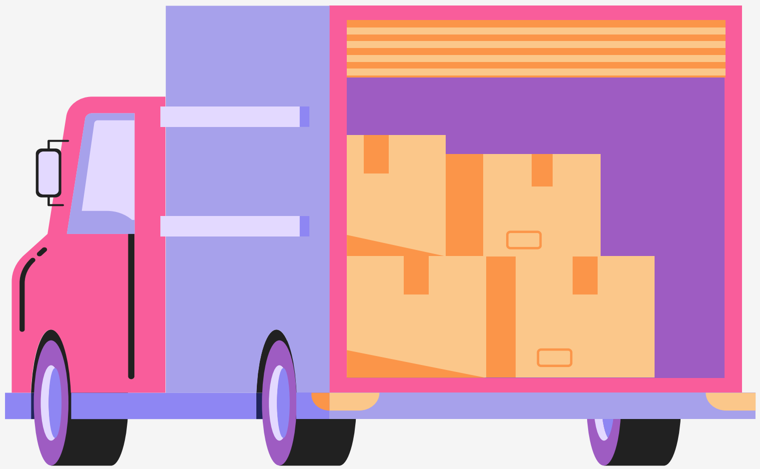
Our scenario

A set **I** of items to be delivered to a set V of costumers.



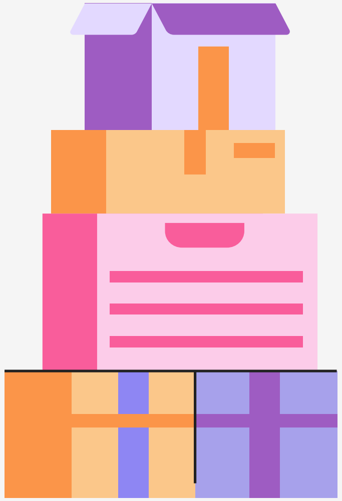
A **platform** receives a price p_i for each item to be delivered.

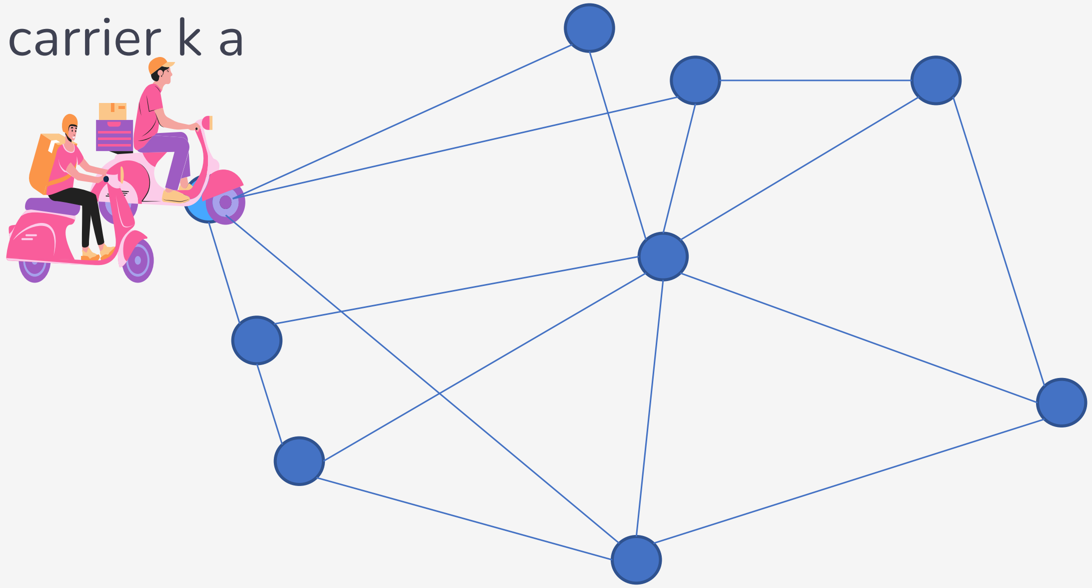
Given a set K of potential **carriers** (e.g., occasional drivers), the platform searches for carriers that can deliver subsets of **I**, and **pays to carrier k a compensation** $\bar{p}_i^k$ for each delivered item, i.e., for each served customer. Each carrier k pays c_{ij}^k to go from costumer i to costumer j.



Our scenario

The **platform** proposes to each carrier k a set of items P_k to serve and a compensation for each item $\bar{p}_i^k$.

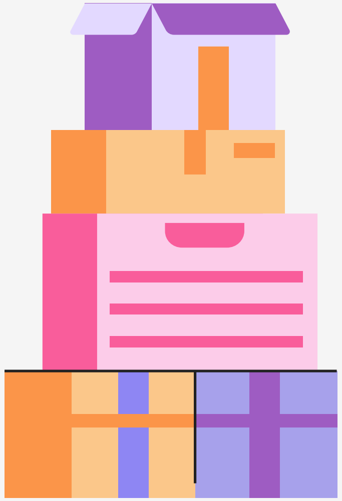


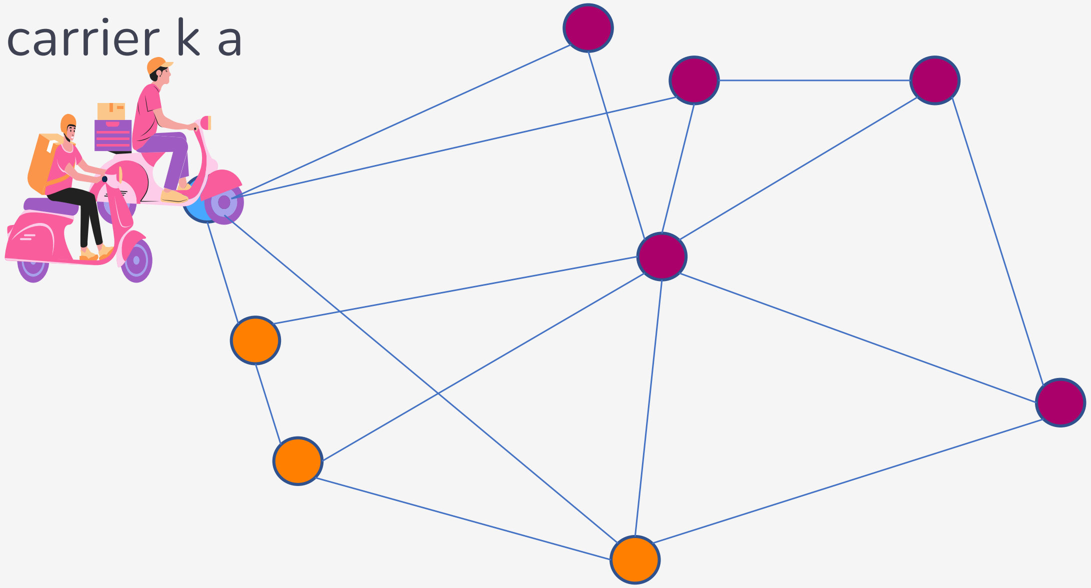


Each **carrier** k receives the proposal, and, based on her net profit, decides on a subset of customers $Q_k \subseteq P_k$ to accept to serve.

Our scenario

The **platform** proposes to each carrier k a set of items P_k to serve and a compensation for each item $\bar{p}_i^k$.





Each **carrier** k receives the proposal, and, based on her net profit, decides on a subset of customers $Q_k \subseteq P_k$ to accept to serve.

Optimization problem

How the **platform** maximizes its own profit if it has no direct control over the **carriers**?

Bilevel Programming

Bilevel programming

$$\text{“}\min_{x \in \mathcal{X}}\text{”} \; F(x, y)$$

$$s.t. \; G(x, y) \leq 0$$

$$y \in \arg\min_{y' \in \mathcal{Y}} \{ f(x, y') \mid g(x, y') \leq 0 \}$$

Optimistic approach

$$\min_{x \in \mathcal{X}} \min_{y \in S(x)} F(x, y)$$

Pessimistic approach

$$\min_{x \in \mathcal{X}} \max_{y \in S(x)} F(x, y)$$

Reformulations

Value function reformulation

One way to reformulate the bilevel problem is considering the so-called *value function* of the lower-level problem:

$$\varphi(x) = \min_{y'} \{f(x, y') | g(x, y') \leq 0\},$$

obtaining the following single-level reformulation:

$$\begin{aligned} \min_{x,y} \; & F(x, y) \\ s.t. \; & G(x, y) \leq 0 \\ & g(x, y) \leq 0 \\ & f(x, y) \leq \varphi(x) \\ & x \in \mathcal{X}, y \in \mathcal{Y}. \end{aligned}$$

The bilevel formulation

The bilevel framework

In our problem, the platform acts as the leader. The carriers act as the |K| followers.

Parameters and sets:

- V set of customers to be served (or equivalently items to be delivered)
- K set of independent carriers
- p_i price the customer i pays to the platform
- c_{ij}^k routing cost that carrier k pays to go from customer i to customer j
- b^k maximum number of customers carrier k can serve

Decisions:

- The **platform** proposes to each carrier k a set of items P_k to serve and a compensation for each item $\bar{p}_i^k$.
- Each **carrier** k receives the proposal, and, based on her net profit, decides on a subset of customers $Q_k \subseteq P_k$ to accept to serve.

The bilevel framework

The goal of each carrier k is maximizing her net profit:

$$\sum_{i \in Q_k} \bar{p}_i^k - \sum_{i \in Q_k} \sum_{j \in Q_k} c_{ij}^k.$$

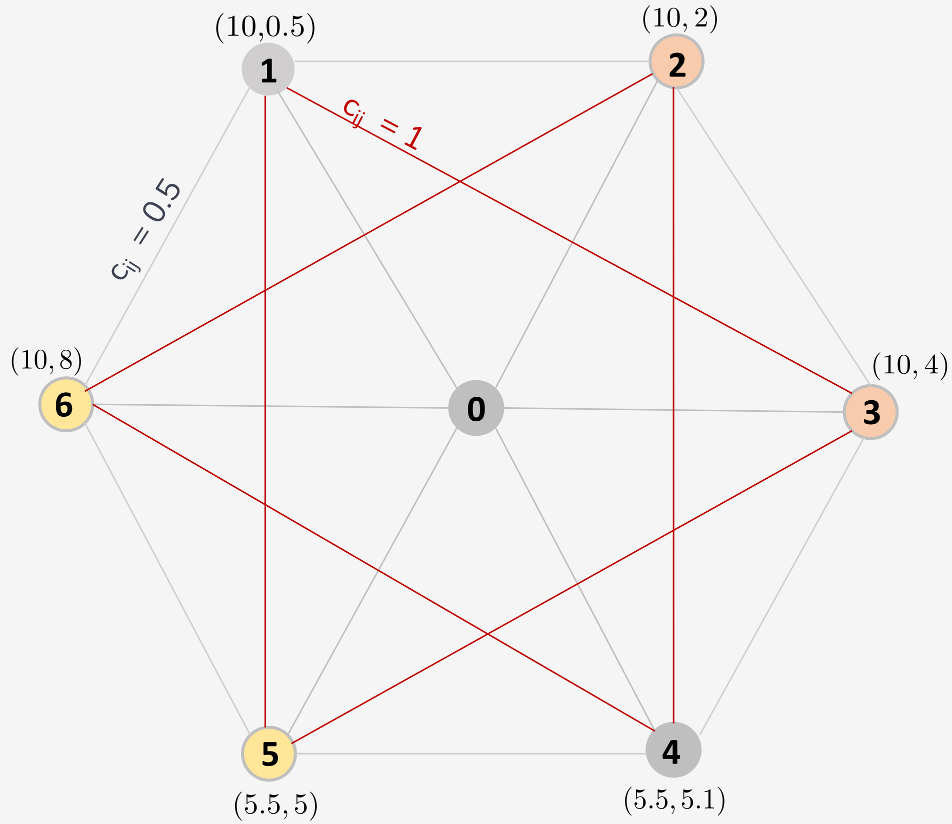
The goal of the leader is making a call to the carriers, so as to maximize his profit, which is defined as:

$$\sum_{k \in K} \sum_{i \in Q_k} (p_i - \bar{p}_i^k).$$

Do we really need bilevel optimization?

- $V = \{1, 2, 3, 4, 5, 6\}$
- $K = \{a, b\}$
- b^k is 2 for all k

For each i:
$(p_i, \bar{p}_i^k)$ for all k

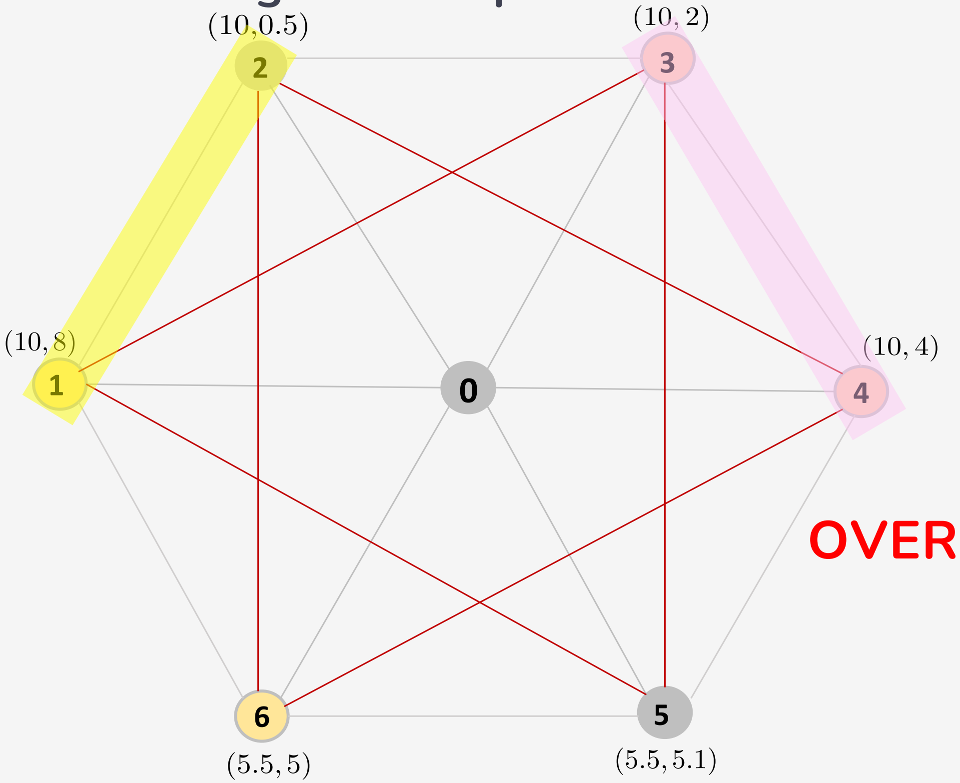


Do we really need bilevel optimization?

In a single-level setting, the leader assigns parcels so as to **maximize** the profit and such that each carrier has a **non-negative net profit**.

The platform assigns items
- 1 and 2 to carrier A
- 3 and 4 to carrier B

Profit = 25.5



In the bilevel setting:
- carrier A accepts only 1
- carrier B accepts both 3 and 4

Profit = 16

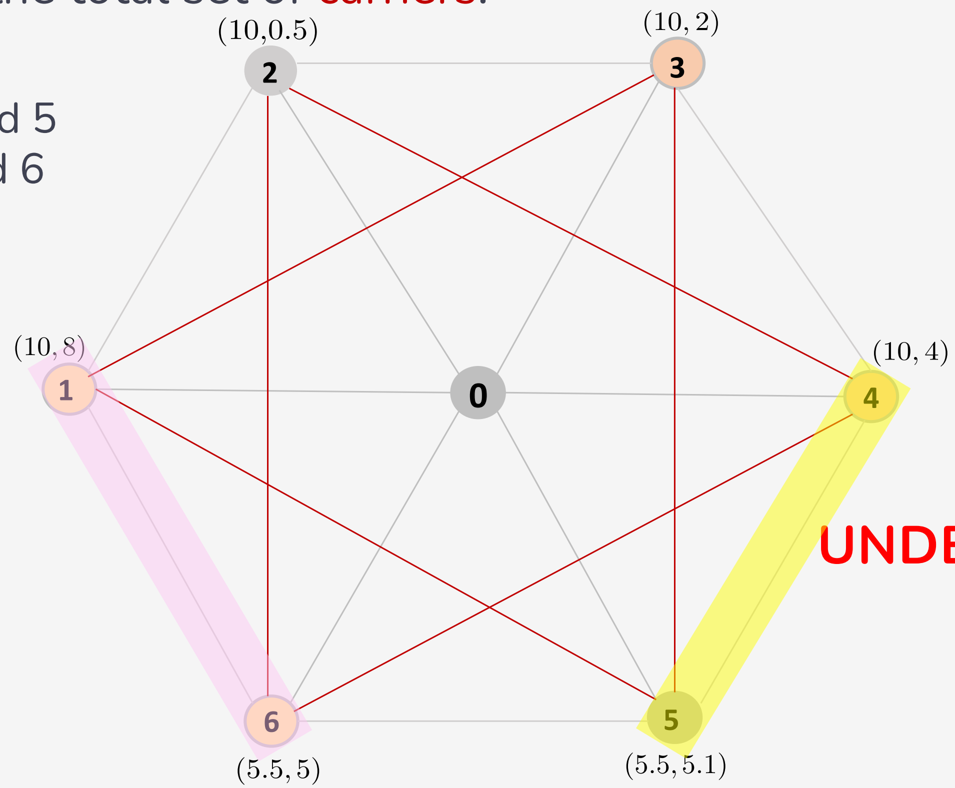
OVERESTIMATION

Do we really need bilevel optimization?

In another single-level setting, we discard the role of the platform and we only maximize the profit of the total set of carriers.

- carrier A serves items 4, and 5
- carrier B serves items 1 and 6

Total profit of the carriers' alliance = 19.1



In the bilevel setting:
- carrier A accepts 4 and 5
- carrier B accepts 1 and 6

Profit = 8.9

In the optimal solution, the leader would assign items 3, 4, 1 and 6.
Profit = 16.5

UNDERESTIMATION

The bilevel framework: compensation decisions

The profit of the leader is:

$$\sum_{k \in K} \sum_{i \in Q_k} (p_i - \boxed{\bar{p}_i^k}).$$

We assume that the leader cannot decide on $\bar{p}_i^k$ directly, but has |M| different possible categories of profit margins that can choose to gain for each item.

The profit margin m = $\frac{(p_i - \bar{p}_i^k)}{p_i}$.

Thus, for an item i and a vehicle k, the net profit $(p_i - \bar{p}_i^k)$ is, for a margin m,

$$p_{mi} := m \cdot p_i$$

Thus, for an item i and a carrier k, the compensation $\bar{p}_i^k$ is, for a margin m,

$$\bar{p}_{mi} = p_i - p_{mi}$$

Variables

Upper-level variables for all $i \in V$:

X^k_{mi} 1 iff the margin selected by the leader for item i and carrier k is m

Lower-level variables for all $i \in V$:

y^k_i 1 iff carrier k **accepts** to serve item i, i.e. if $i \in Q_k$

z^k_{ij} 1 iff arc (i, j) is traversed by carrier k to deliver items i and j from Q_k

Objective functions

Being $m = \frac{(p_i - \bar{p}_i^k)}{p_i}$ and $p_{mi} = m p_i = p_i - \bar{p}_i^k$, and $\bar{p}_{mi} = p_i - p_{mi}$:

- The goal of the leader is making a call to the carriers, so as to maximize his net profit:

$$\sum_{k \in K} \sum_{i \in Q_k} (p_i - \bar{p}_i^k) = \sum_{k \in K} \sum_{m \in M} \sum_{i \in V} p_{mi} X_{mi}^k y_i^k.$$

- The goal of each follower k is maximizing her net profit:

$$\sum_{i \in Q_k} \bar{p}_i^k - \sum_{i \in Q_k} \sum_{j \in Q_k} c_{ij}^k = \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} X_{mi}^k y_i^k - \sum_{(i,j) \in A} c_{ij}^k z_{ij}^k.$$

The bilevel formulation

$$\max_{X,y} \sum_{k\in K}\sum_{i\in V}\sum_{m\in M} p_{mi} X_{mi}^{k} y_i^{k}$$

$$\text{s.t.} \sum_{k\in K}\sum_{m\in M} X_{mi}^{k} \leq 1 \qquad \forall\, i \in V \setminus \{0\}$$

$$y^k \in S_\Phi^k(X^k) \qquad \forall\, k \in K$$

$$y^k, X_m^k \in \{0,1\}^n \qquad \forall\, m \in M, k \in K$$

where $S_\Phi^k(X^k)$ is the set of optimal solutions of the k-th follower problem.

Follower's problem (the Profitable Tour Problem)

The carrier (follower) k problem for a given $\tilde{X}^k$ is formulated as:

$$
\begin{aligned}
\Phi^k(\tilde{X}^k) = \max_{y,z} & \sum_{i\in V}\sum_{m\in M} \bar{p}_{mi}\tilde{X}^k_{mi}y^k_i - \sum_{(i,j)\in A} c^k_{ij}z^k_{ij} \\
\text{s.t.} \quad & y^k_i \leq \sum_{m\in M} \tilde{X}^k_{mi} \qquad \forall\, i \in V \\
& \sum_{i\in V} y^k_i \leq b^k \\
& (y^k, z^k) \text{ is a route} \\
& y^k \in \{0,1\}^{n+1}, z^k \in \{0,1\}^{|A|}
\end{aligned}
$$

Optimistic setting

We assume that we are in an optimistic setting, i.e., for a given choice of $\tilde{X}^k$, if follower k has multiple optimal responses determined by different sets Q_k of items to be delivered, she will accept to deliver the items which are more favorable to the leader:

$$\tilde{Q}_k = \arg\max_{Q_k} \{ \sum_{i \in Q_k} p_{mi} \tilde{X}^k_{mi} \; : \; Q_k \in S^k_\Phi(\tilde{X}^k) \}.$$

Dealing with bilinear terms

We can linearize this bilevel MINLP using the McCormick's inequalities. We introduce additional binary variables w_{mi}^k defined as $X_{mi}^k y_i^k$ and adjoin the following inequalities to the upper-level model:

$$
\begin{aligned}
X_{mi}^k + y_i^k \leq w_{mi}^k + 1 \quad & \forall\, m \in M, i \in V, k \in K \\
w_{mi}^k \leq X_{mi}^k \quad & \forall\, m \in M, i \in V, k \in K \\
w_{mi}^k \leq y_i^k \quad & \forall\, m \in M, i \in V, k \in K
\end{aligned}
$$

Linearized bilevel model

$$
\begin{aligned}
\max_{X,w,y} \quad & \sum_{k\in K}\sum_{i\in V}\sum_{m\in M} p_{mi} w_{mi}^k \\
\text{s.t.} \quad & \sum_{k\in K}\sum_{m\in M} X_{mi}^k \leq 1 && \forall\, i \in V \setminus \{0\} \\
& y^k \in S_\Phi^k(X^k) && \forall\, k \in K \\
& \text{McCormick Inequalities} \\
& X_m^k, w_m^k \in \{0,1\}^n && \forall\, m \in M, k \in K \\
& y^k \in \{0,1\}^{n+1} && \forall\, k \in K
\end{aligned}
$$

Single-level reformulations

Value function reformulation

$$
\begin{aligned}
\max_{X,w,y,z} \quad & \sum_{k\in K}\sum_{i\in V}\sum_{m\in M} p_{mi} w_{mi}^k \\
\text{s.t.} \quad & \sum_{k\in K}\sum_{m\in M} X_i^k \le 1 && \forall\, i\in V\setminus\{0\} \\
& \sum_{i\in V} y_i^k \le b^k && \forall\, k\in K \\
& \text{McCormick ineq.} && \forall m\in M, i\in V, k\in K \\
& y_i^k \le \sum_{m\in M} X_{mi}^k && \forall\, i\in V, k\in K \\
& (y^k, z^k) \text{ is a route} && \forall\, k\in K \\
& \sum_{i\in V}\sum_{m\in M} \bar{p}_{mi} w_{mi}^k - \sum_{(i,j)\in A} c_{ij}^k z_{ij}^k \ge \Phi^k(X^k) && \forall\, k\in K
\end{aligned}
$$

Value function reformulation

$$\max_{X,w,y,z} \quad \sum_{k\in K}\sum_{i\in V}\sum_{m\in M} p_{mi} w^k_{mi}$$

$$\text{s.t.} \quad \sum_{k\in K}\sum_{m\in M} X^k_i \le 1 \qquad \forall\, i \in V \setminus \{0\}$$

$$\sum_{i\in V} y^k_i \le b^k \qquad \forall\, k \in K$$

$$\text{McCormick ineq.} \qquad \forall m \in M, i \in V, k \in K$$

$$y^k_i \le \sum_{m\in M} X^k_{mi} \qquad \forall\, i \in V, k \in K$$

$$(y^k, z^k) \text{ is a route} \qquad \forall\, k \in K$$

$$\sum_{i\in V}\sum_{m\in M} \bar{p}_{mi} w^k_{mi} - \sum_{(i,j)\in A} c^k_{ij} z^k_{ij} \ge \Phi^k(X^k) \qquad \forall\, k \in K$$

Value function reformulation

$$
\begin{aligned}
\max_{X,w,y,z} \quad & \sum_{k\in K}\sum_{i\in V}\sum_{m\in M} p_{mi} w_{mi}^k & \\
\text{s.t.} \quad & \sum_{k\in K}\sum_{m\in M} X_i^k \le 1 & \forall\, i \in V \setminus \{0\} \\
& \sum_{i\in V}\sum_{m\in M} X_{mi}^k \le b^k & \forall\, k \in K \\
& \text{McCormick ineq.} & \forall m \in M, i \in V, k \in K \\
& y_i^k \le \sum_{m\in M} X_{mi}^k & \forall\, i \in V, k \in K \\
& (y^k, z^k) \text{ is a route} & \forall\, k \in K \\
& \sum_{i\in V}\sum_{m\in M} \bar{p}_{mi} w_{mi}^k - \sum_{(i,j)\in A} c_{ij}^k z_{ij}^k \ge \Phi^k(X^k) & \forall\, k \in K
\end{aligned}
$$

Proposition

$$
\begin{aligned}
\Phi^k(\tilde{X}^k) = \ \max_{y,z} \ & \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} \tilde{X}^k_{mi} y^k_i - \sum_{(i,j) \in A} c^k_{ij} z^k_{ij} \\
\text{s.t.} \quad & y^k_i \leq \sum_{m \in M} \tilde{X}^k_{mi} && \forall \ i \in V \\
& \sum_{i \in V} y^k_i \leq b^k \\
& (y^k, z^k) \text{ is a route} \\
& y^k \in \{0,1\}^{n+1}, z^k \in \{0,1\}^{|A|}
\end{aligned}
$$

Proposition

There always exists an optimal solution of the following problem, which is also optimal for $\Phi^k(\tilde{X}^k)$*:*

$$\begin{aligned} \bar{\Phi}^k(\tilde{X}^k) = \max_{y,z} & \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} \tilde{X}^k_{mi} y^k_i - \sum_{(i,j) \in A} c^k_{ij} z^k_{ij} \\ \text{s.t.} \quad & (y^k, z^k) \text{ is a route} \\ & y^k \in \{0,1\}^{n+1}, z^k \in \{0,1\}^{|A|} \end{aligned}$$

Single-level reformulation

Let P_{ext}^k denote the set of all the extreme points (y^k, z^k) of the convex hull of the profitable tour feasible solutions determined by constraints "(y^k, z^k) is a route". It holds:

$$\Phi^k(X^k) = \max_{(\hat{y}^k, \hat{z}^k) \in P_{ext}^k} \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} X_{mi}^k \hat{y}_i^k - \sum_{(i,j) \in A} c_{ij}^k \hat{z}_{ij}^k$$

Thus, by replacing value function constraint for each k

$$\sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} w_{mi}^k - \sum_{(i,j) \in A} c_{ij}^k z_{ij}^k \geq \Phi^k(X^k)$$

Single-level reformulation

Let P^k_{ext} denote the set of all the extreme points (y^k, z^k) of the convex hull of the profitable tour feasible solutions determined by constraints "(y^k, z^k) is a route". It holds:

$$\Phi^k(X^k) = \max_{(\hat{y}^k, \hat{z}^k) \in P^k_{ext}} \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} X^k_{mi} \hat{y}^k_i - \sum_{(i,j) \in A} c^k_{ij} \hat{z}^k_{ij}$$

Thus, by replacing value function constraint for each k

$$(\star) \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} w^k_{mi} - \sum_{(i,j) \in A} c^k_{ij} z^k_{ij} \geq \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} X^k_{mi} \hat{y}^k_i - \sum_{(i,j) \in A} c^k_{ij} \hat{z}^k_{ij}, \quad \forall (\hat{y}^k, \hat{z}^k) \in P^k_{ext}$$

we obtain a single-level reformulation of our problem.

Single-level reformulation

Let P_{ext}^k denote the set of all the extreme points (y^k, z^k) of the convex hull of the profitable tour feasible solutions determined by constraints "(y^k, z^k) is a route". It holds:

$$\Phi^k(X^k) = \max_{(\hat{y}^k, \hat{z}^k) \in P_{ext}^k} \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} X_{mi}^k \hat{y}_i^k - \sum_{(i,j) \in A} c_{ij}^k \hat{z}_{ij}^k$$

Thus, by replacing value function constraint for each k

$$(\star) \quad \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} w_{mi}^k - \sum_{(i,j) \in A} c_{ij}^k z_{ij}^k \geq \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} X_{mi}^k \hat{y}_i^k - \sum_{(i,j) \in A} c_{ij}^k \hat{z}_{ij}^k, \quad \forall\, (\hat{y}^k, \hat{z}^k) \in P_{ext}^k$$

Exponentially many!

we obtain a single-level reformulation of our problem.

Separation procedure

- Relax constraints $(\star)$ from the bilevel problem, finding solution $(\tilde{X}^k, \tilde{w}^k, \tilde{y}^k, \tilde{z}^k)$
- Solve the profitable tour problem (lower level) for $X^k = \tilde{X}^k$ for each k, obtaining solution $(\hat{y}^k, \hat{z}^k)$ with optimal value $\hat{\Phi}^k$
- If it exists a k such that

$$\sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} \tilde{w}^k_{mi} - \sum_{(i,j) \in A} c^k_{ij} \tilde{z}^k_{ij} < \hat{\Phi}^k,$$

add the cut

$$\sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} w^k_{mi} - \sum_{(i,j) \in A} c^k_{ij} z^k_{ij} \geq \sum_{i \in V} \sum_{m \in M} \bar{p}_{mi} X^k_{mi} \hat{y}^k_i - \sum_{(i,j) \in A} c^k_{ij} \hat{z}^k_{ij}$$

to the master problem. Otherwise, the obtained solution is optimal for the original bilevel formulation.

Projection of z variable

Projecting out the z variable

Since the platform profit is not depending on the route followed by each carrier, there is no need to consider z variables at the master level:

$$
\begin{aligned}
\max_{X,w,y,z} \quad & \sum_{k\in K}\sum_{i\in V}\sum_{m\in M} p_{mi} w_{mi}^k \\
\text{s.t.} \quad & \sum_{k\in K}\sum_{m\in M} X_i^k \le 1 && \forall\, i \in V \setminus \{0\} \\
& \sum_{i\in V} y_i^k \le b^k && \forall\, k \in K \\
& \text{McCormick ineq.} && \forall m \in M, i \in V, k \in K \\
& y_i^k \le \sum_{m\in M} X_{mi}^k && \forall\, i \in V, k \in K \\
& (y^k, z^k) \text{ is a route} && \forall\, k \in K \\
& \sum_{i\in V}\sum_{m\in M} \bar{p}_{mi} w_{mi}^k - \sum_{(i,j)\in A} c_{ij}^k z_{ij}^k \ge \Phi^k(X^k) && \forall\, k \in K
\end{aligned}
$$

Projecting out the z variable

Since the platform profit is not depending on the route followed by each carrier, there is no need to consider z variables at the master level:

$$
\begin{aligned}
\max_{X,w,y,\theta} \quad & \sum_{k\in K}\sum_{i\in V}\sum_{m\in M} p_{mi} w_{mi}^k & \\
\text{s.t.} \quad & \sum_{k\in K}\sum_{m\in M} X_i^k \leq 1 & \forall\, i \in V \setminus \{0\} \\
& \sum_{i\in V} y_i^k \leq b^k & \forall\, k \in K \\
& \text{McCormick ineq.} & \forall m \in M, i \in V, k \in K \\
& y_i^k \leq \sum_{m\in M} X_{mi}^k & \forall\, i \in V, k \in K \\
& \theta^k \geq c_{TSP}(y^k) & \forall\, k \in K \\
& \sum_{i\in V}\sum_{m\in M} \bar{p}_{mi} w_{mi}^k - \theta^k \geq \Phi^k(X^k) & \forall\, k \in K
\end{aligned}
$$

Projecting out the z variable

Since the platform profit is not depending on the route followed by each carrier, there is no need to consider z variables at the master level:

$$\max_{X,w,y,\theta} \sum_{k\in K}\sum_{i\in V}\sum_{m\in M} p_{mi} w^k_{mi}$$

$$\text{s.t.} \quad \sum_{k\in K}\sum_{m\in M} X^k_i \leq 1 \qquad \forall\, i \in V \setminus \{0\}$$

$$\sum_{i\in V} y^k_i \leq b^k \qquad \forall\, k \in K$$

$$\text{McCormick ineq.} \qquad \forall m \in M, i \in V, k \in K$$

$$y^k_i \leq \sum_{m\in M} X^k_{mi} \qquad \forall\, i \in V, k \in K$$

$$(\star\star) \quad \theta^k \geq c_{TSP}(y^k) \qquad \forall\, k \in K$$

Exponentially many!

$$(\star) \quad \sum_{i\in V}\sum_{m\in M} \bar{p}_{mi} w^k_{mi} - \theta^k \geq \Phi^k(X^k) \qquad \forall\, k \in K$$

Heuristic warm-start procedure

Heuristic algorithm

- We solve the problem without margin decision, setting the compensation to $\bar{p}_i^k = (1 - m_{min})p_i$ for all i and k.
- We obtain the optimal solution in terms of assignment of the leader $\hat{x}^k$, and acceptance and routing decisions of the followers $\hat{T}^k$.
- We solve the "only-pricing problem":

$$
\begin{aligned}
\max_{X} \; & \sum_{k \in K} \sum_{m \in M} \sum_{i \in V(\hat{T}^k)} p_{mi} X_{mi}^k \\
\text{s.t.} \; & \sum_{m \in M} X_{mi}^k = \hat{x}_i^k && \forall\, i \in V, k \in K \\
& \sum_{m \in M} \sum_{i \in V(\hat{T}^k)} p_{mi} X_{mi}^k \leq \sum_{i \in V(\hat{T}^k)} p_i - C(\hat{T}^k) && \forall\, k \in K,
\end{aligned}
$$

- We obtain the optimal solution in terms of margin decisions $\check{X}$.
- We solve the problem without margin decision, setting the compensation to $\bar{p}_i^k = p_i - \sum_{m \in M} \check{X}_{mi}^k p_{mi}$ for all i and k.

Computational results

Instances

- **Chao**'s instances for the OP: number of customers ranging from 21 to 66
- **Solomon** instances for the VRPTW: number of customers ranging from 20 to 35
  - 2, 3 or 4 carriers
  - Different margins sets: {0.2,0.5}; {0.5,0.9}; {0.2,0.5,0.8}; {0.5,0.7,0.9} * these are the margins to the platform
  - Time limit 1hour
  - CPLEX 22.1.0.0

	Heuristic	Model (BPMD)								Model (BPMD-z)							
	LB$_h$	#opt	LB	UB	gap	time	septime	#sep	#nodes	#opt	LB	UB	gap	time	septime	#sep	#nodes
Chao instances																	
{0.2, 0.5}	1076	15	1076	1076	0.00	6.3	0.0	1	0	15	1076	1076	0.00	6.8	1.6	3	0
{0.5, 0.9}	1876	0	1892	1937	2.36	3600	1254	384	193346	0	1907	1937	1.60	3600	2541	4472	121306
{0.2, 0.5, 0.8}	1716	11	1719	1722	0.18	1477	723	309	36777	11	1719	1722	0.17	1337	1037	2337	19481
{0.5, 0.7, 0.9}	1881	0	1893	1937	2.33	3600	1310	430	170169	0	1911	1937	1.34	3600	2397	4844	143357
Solomon instances																	
{0.2, 0.5}	672	9	675	676	0.20	904	114	98	50971	9	676	676	0.08	986	827	1409	18437
{0.5, 0.9}	770	2	898	1003	9.04	3163	245	377	579250	0	875	1068	17.0	3600	980	6279	435079
{0.2, 0.5, 0.8}	790	5	945	1010	5.25	2584	145	235	571582	0	918	1059	12.4	3600	981	5947	434403
{0.5, 0.7, 0.9}	773	5	929	1015	6.92	2714	218	342	579739	0	915	1083	14.5	3600	948	5051	489491

Solutions structure

	Leader's Profit	%high	%medium	%low	%served	time
R20_2						
{0.2, 0.5}	487.5	100	-	0	100	0.1
{0.5, 0.9}	675.7	41.2	-	58.8	89.5	456
{0.2, 0.5, 0.8}	691.8	68.8	31.2	0	84.2	70
{0.5, 0.7, 0.9}	731.8	33.3	44.4	22.2	90	223
R20_3						
{0.2, 0.5}	487.5	100	-	0	100	0.2
{0.5, 0.9}	661.3	41.2	-	58.8	89.5	1490
{0.2, 0.5, 0.8}	675	62.5	37.5	0	84.2	431
{0.5, 0.7, 0.9}	705.6	22.2	50	27.8	90	745

Conclusions

- The problem becomes harder to solve when margins are such that mix of them is made in the optimal solution
- Solomon’s instances are harder – probably related to the geography of customers
- Projecting out z variables pays off on Chao’s instances, but not on Solomon’s ones

Thank you! ☺

Any questions?

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