Improving Strong-Branching Decisions With Additional Information

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Incorporating Primal Information

Strong Branching:

LP gains for a var i on branch j, Δ_i^j

SB score$(i) = \Delta_i^+ \cdot \Delta_i^-$

	Δ^+	Δ^-	SB
x_1	1.8	0.6	1.08
x_2	1.1	0.9	0.99

Efficacious Strong Branching (Eff-SB):

Primal-dual gap at node = Δ_{p-d}

Efficacious gains, $q_i = \min\{\Delta_i, \Delta_{p-d}\}$

Eff score$(i) = q_i^+ \cdot q_i^-$

	q^+	q^-	Eff-SB
x_1	1.	0.6	0.6
x_2	1.	0.9	0.9

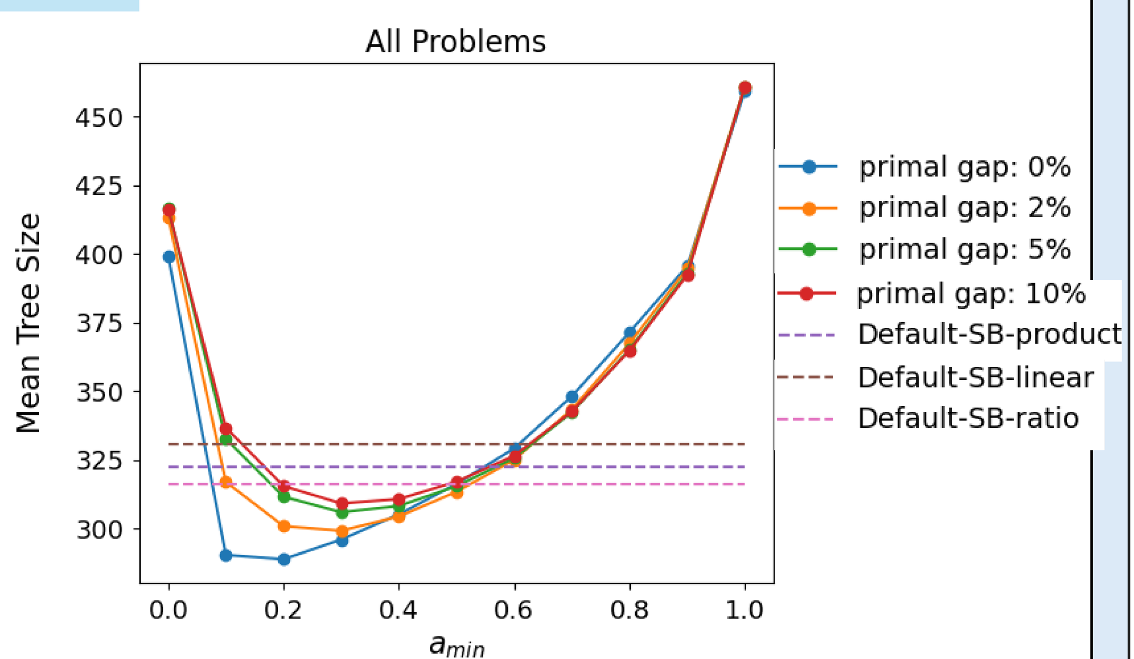
Optimizing score for Eff-SB

Applying parameterization,

score $= (q_{min})^{a_{min}} \cdot (q_{max})^{a_{max}}$

$q_{min/max} = \min/\max(q_i^+, q_i^-)$

- $a_{max} + a_{min} = 1$
- Primal gap = $\frac{|z_{IP}^* - \hat{z}|}{|z_{IP}^*|}$



- Test bed: 100 instances with 20-200 binaries of 10 classes of problems, Multi-row Packing, Covering and Mixed IPs, Portfolio Optimization (CCP), Fixed charge network flow, Constrained lot-sizing variants and Weighted Matching, Stable Set, Set Coverage
- No Presolve, Cuts, etc; Optimal objective value provided

Default SB Tree size	Tree size Reduction – Eff SB				
	Primal Gap :	0%	2%	5%	10%
322.4		8.2%	7.2%	5.1%	4.2%

MIPLIB Experiments: Eff SB score = $(q_{min})^{0.3} \cdot (q_{max})^{0.7}$

- Gurobi Presolve, at most 2000 binary variables after presolve
- 43 instances in test bed
- Default SCIP cuts at root node, Node limit = 10,000, Best-Bound rule

Default SB Gap Closed	Gap Closed - Eff SB				
	Primal Gap :	0%	2%	5%	10%
39.3%		43.7%	42.3%	42.3%	41.5%

Incorporating Structural Information

Motivation: Inherent asymmetry in 0-1 assignments

Packing problems have a higher occurrence of pruning due to infeasibility on 1-branch; likely smaller subtree size as compared to 0-branch

Hypothesis: higher emphasis on q_i^0 as compared to q_i^1 advantageous

Naïve Structural Score

Packing structure: score $= (q_i^-)^{0.15}(q_{min})^{0.3} \cdot (q_{max})^{0.7}$

Covering structure: score $= (q_i^+)^{0.15}(q_{min})^{0.3} \cdot (q_{max})^{0.7}$

Special Case: Cardinality Constrained ($\sum_{i=1}^n x_i \leq k$)

At a node, n free variables, k cardinality limit

Sub-tree size estimator recursion,

$$\hat{T}(n,k) = \hat{T}(n-1,k) + \hat{T}(n-1,k-1) + 1$$

Satisfied approximately by $\hat{T}(n,k) = \binom{n}{k}$

Fractional tree size at branches 0 and 1: $\frac{\hat{T}(n-1,k)}{\hat{T}(n,k)} = \frac{n-k}{n}$, $\frac{\hat{T}(n-1,k-1)}{\hat{T}(n,k)} = \frac{k}{n}$

$$\text{score}(i) = (q_{i,0})^{\frac{n-k}{n}} \times (q_{i,1})^{\frac{k}{n}} \times (q_{i,min})^{0.3} \times (q_{i,max})^{0.7}$$

Computationally Estimating Asymmetry

I. Constraint restrictiveness ($ax \leq b$)

For problems with explicit packing or covering structure

- Higher asymmetry if b is relatively small or a_j is relatively large

$$r_j = (b - a_j)/\sum_{i \neq j} a_i$$

- Individual score function for each variable at a node

II. Information from Partial Trees

For general problems with implicit or undetermined structure

a. Clauses (leaf nodes) –

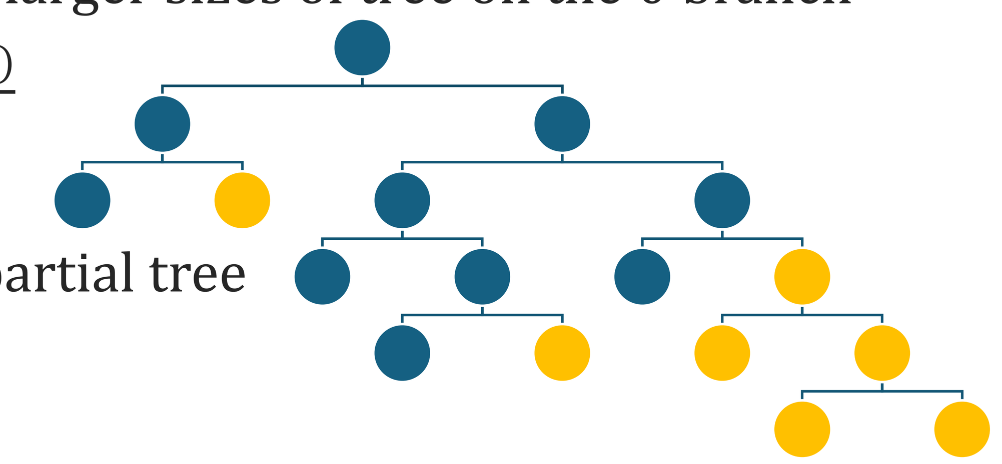
More variables fixed to 0 imply larger sizes of tree on the 0 branch

$$\frac{(\#\text{ 0 assignments} - \#\text{ 1 assignments})}{\#\text{ clauses} \cdot \#\text{ binary vars}}$$

b. Sub-tree sizes –

Over every *solved* node of the partial tree

$$\text{Mean}\left(\frac{\text{size on 0 branch}}{\text{size on 1 branch}}\right)$$



Computational Experiments : On 20 randomly generated instances, providing optimal obj value, without presolve, cuts, etc

Set Based IPs | **Random IPs** | **Mixed-integer**