The Terminator: An Integration of Inner and Outer Approximations for Solving Wasserstein Distributionally Robust Chance Constrained Programs via Variable Fixing

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2024 Mixed Integer Programming Workshop
Georgia Tech College of Engineering
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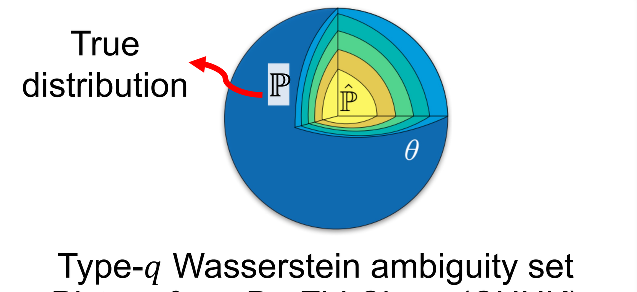
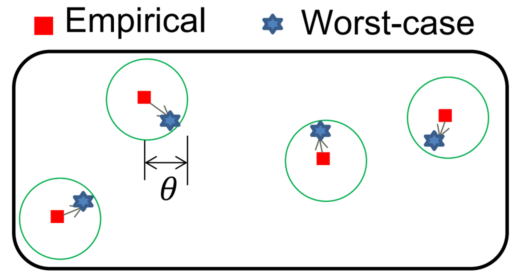
Introduction

Distributionally Robust Chance Constrained Programs (DRCCPs) aim to identify optimal decisions that satisfy uncertain constraints at a preset level of risk, across all probability distributions within Wasserstein ambiguity set

$$v^* = \min_{x} \left\{ c^\top x : \inf_{\mathbb{P}\in\mathcal{P}_q} \mathbb{P}\{\tilde{\xi} : g(x,\tilde{\xi}) \le 0\} \ge 1-\varepsilon, x \in \mathcal{X} \right\}$$



Type-q Wasserstein ambiguity set
Picture from Dr. Zhi Chen (CUHK)



Type-∞ Wasserstein ambiguity set

Difficulties of DRCCPs: The feasible region is nonconvex

Computation Challenges for DRCCPs

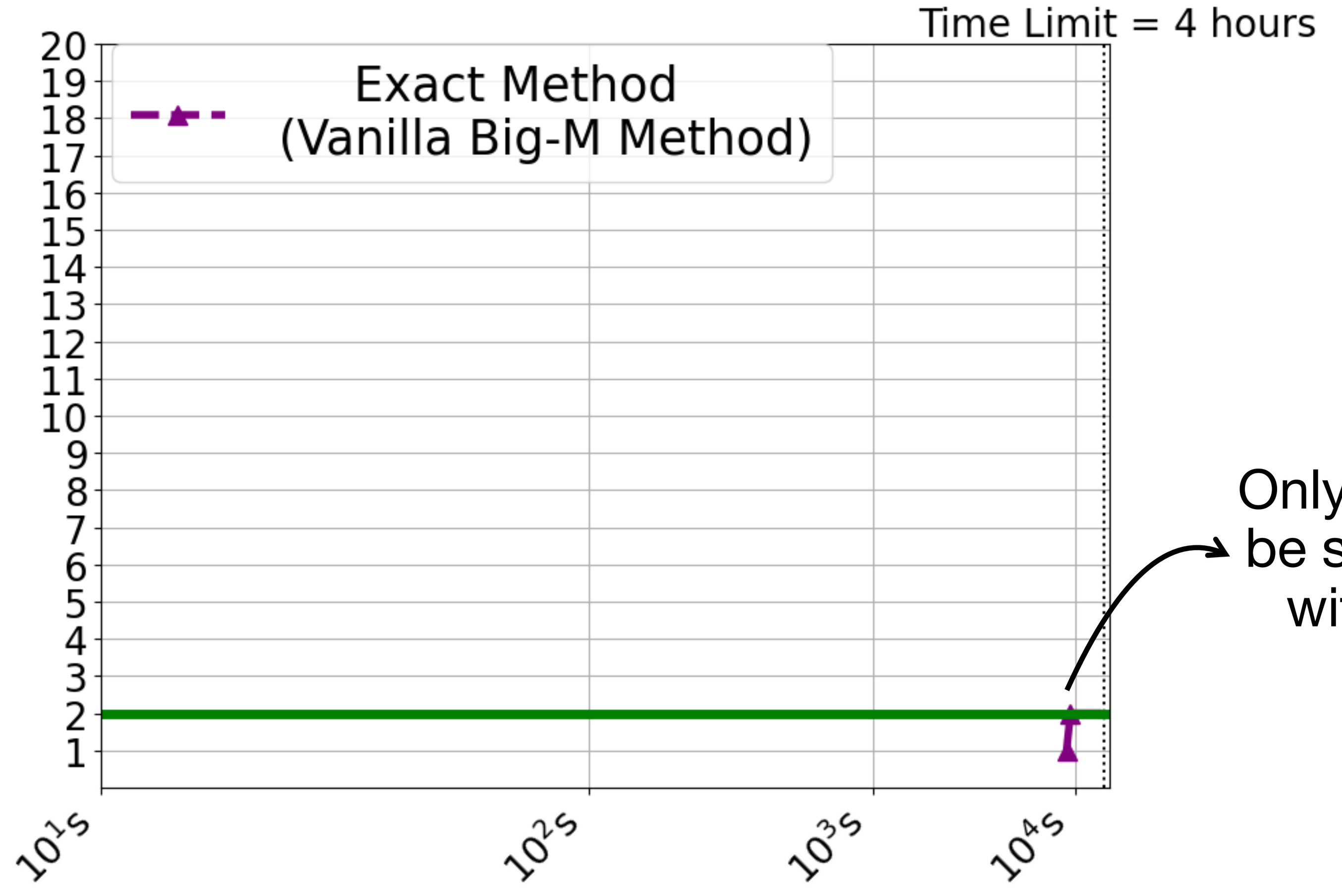
Solving DRCCPs to optimally is time-consuming

Example:

$$v^* = \min_{x} \left\{ c^\top x : \inf_{\mathbb{P}\in\mathcal{P}_1} \mathbb{P}\{\tilde{\xi} : \tilde{\xi}^\top x - b \le 0\} \ge 1-\varepsilon, x \in [0,1]^n \right\}$$

Run 20 instances from Song et al. (2014) and set 4-hour time limit for each instance

Vertical axis: The number of instances solved to optimality



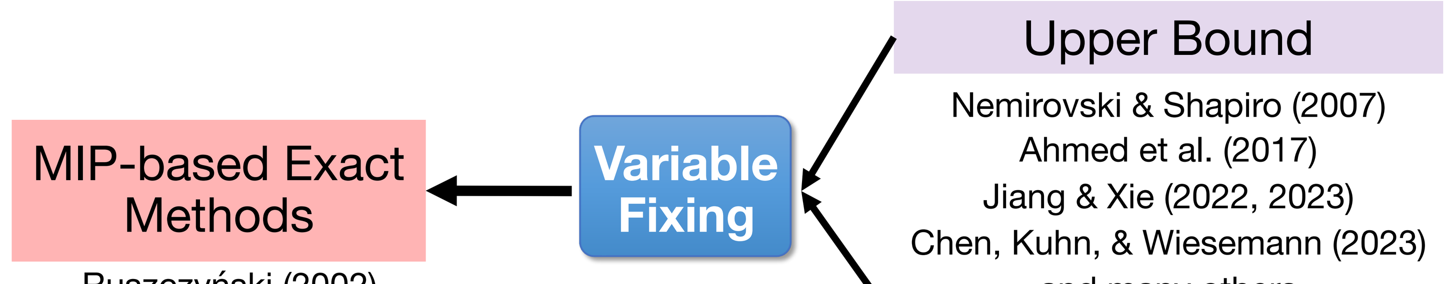
Only two instances can be solved to optimality within the time limit

Motivation: Small Gaps in DRCCPs

Jiang & Xie (2022, 2023) provide high-quality upper/lower bounds

Instance	Improved Upper Bound $\frac{\lvert v^U - v^*\rvert}{v^*}$	Improved Lower Bound $\frac{\lvert v^L - v^*\rvert}{v^*}$
1	0.3%	0.8%
2	0.4%	0.7%
3	0.3%	0.9%
4	0.2%	0.8%

Generate effective **optimality cuts** using easily computable upper/lower bounds



MIP-based Exact Methods: Ruszczyński (2002), Luedtke & Ahmed (2008), Xie (2021), Ji & Lejeune (2021), Ho-Nguyen et al. (2022, 2023), Chen, Kuhn, & Wiesemann (2022) and many others

Upper Bound: Nemirovski & Shapiro (2007), Ahmed et al. (2017), Jiang & Xie (2022, 2023), Chen, Kuhn, & Wiesemann (2023) and many others

Lower Bound: Song, Luedtke, & Küçükyavuz (2014), Ahmed et al. (2017), Xie (2021) and many others

Big-M Exact Method with Regular CCPs ($\theta = 0$ in Wasserstein)

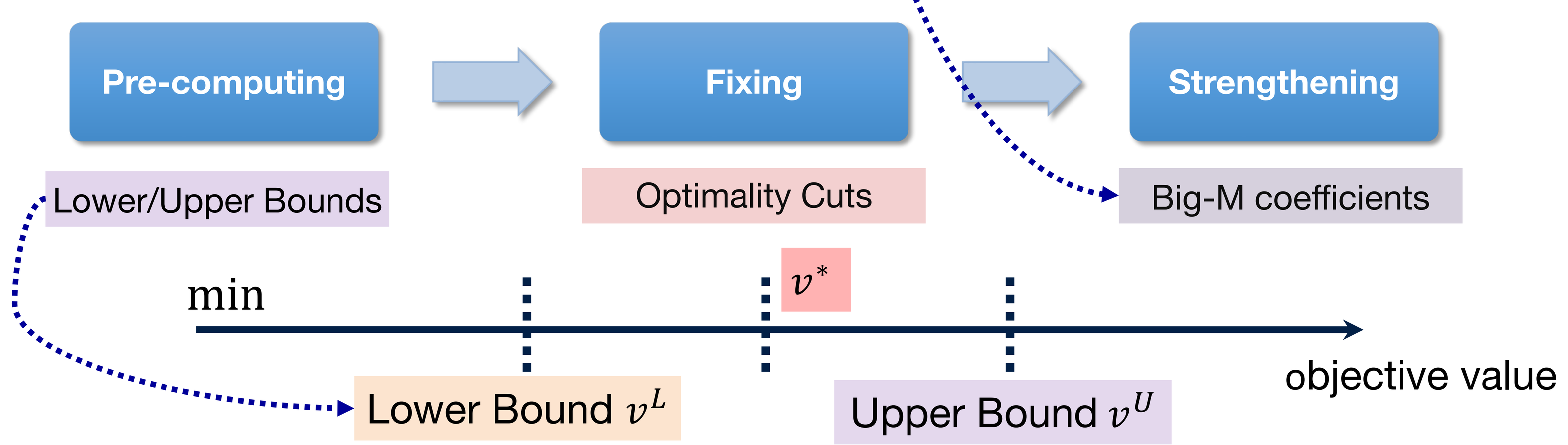
$$v^* = \min_{x} \left\{ c^\top x : \frac{1}{N}\sum_{i\in[N]} \mathbb{I}\{g(x,\hat{\xi}^i) \le 0\} \ge 1-\varepsilon, x \in \mathcal{X} \right\}$$

Our results hold for DRCCPs ($\theta > 0$)

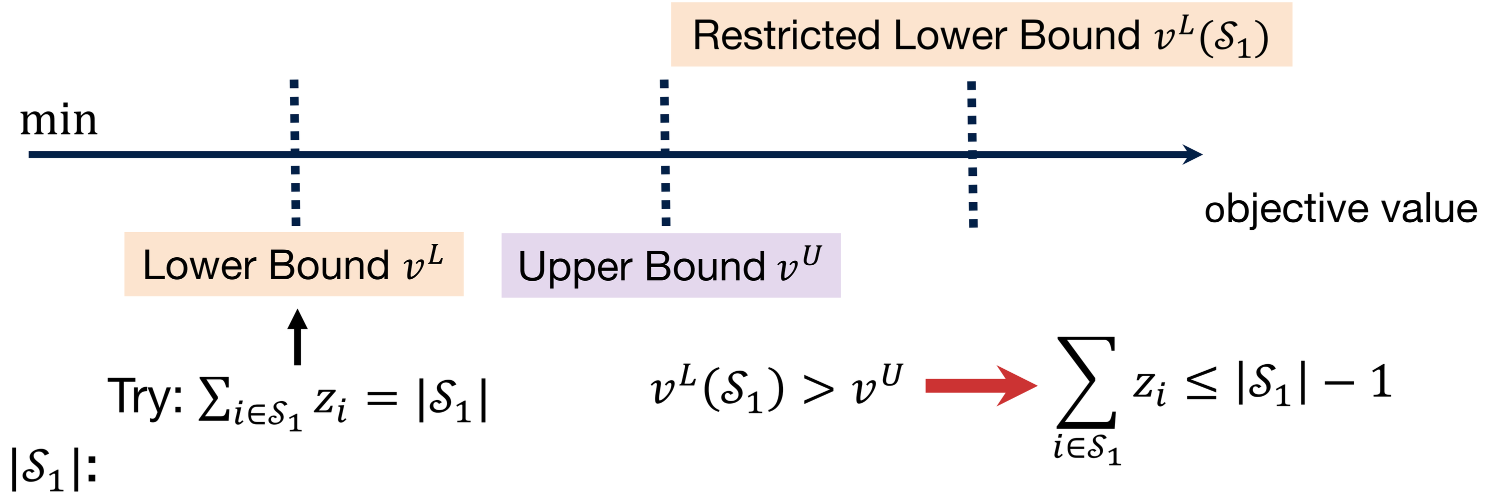
Binary variable $z_i = 1$ if and only if ith scenario is satisfied with N scenarios

$$v^* = \min_{x\in\mathcal{X}, z} \left\{ c^\top x : \begin{array}{l} g(x,\hat{\xi}^i) \le M_i(1-z_i), \forall i \in [N] \\ \frac{1}{N}\sum_{i\in[N]} z_i \ge 1-\varepsilon, z \in \{0,1\}^N \end{array} \right\}$$

Variable Fixing Procedure for Regular CCPs



Fixing Procedure/Optimality Cuts Generation



Try $\sum_{i\in\mathcal{S}_1} z_i = |\mathcal{S}_1|$:

Theorem 1

For set $\mathcal{S}_1 \subset [N]$, if $v^L(\mathcal{S}_1) > v^U$, then $\sum_{i\in\mathcal{S}_1} z_i \le |\mathcal{S}_1| - 1$

Try $\sum_{i\in\mathcal{S}_0} z_i = 0$:

Theorem 2

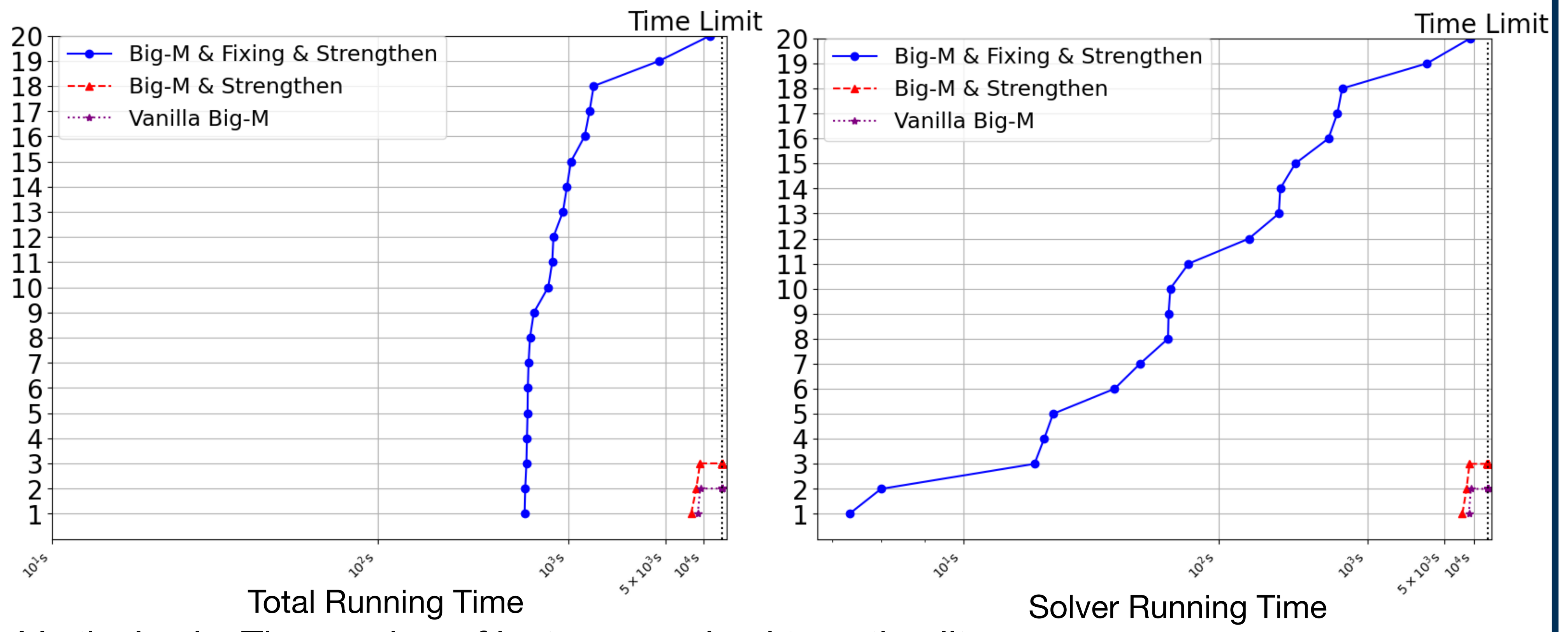
For set $\mathcal{S}_0 \subset [N]$ with $|\mathcal{S}_0| \le \lfloor N\varepsilon \rfloor$, if $v^L(\mathcal{S}_0) > v^U$, then $\sum_{i\in\mathcal{S}_0} z_i \ge 1$

High-quality upper/lower bounds are important

We provide new lower bound for DRCCPs

Numerical Result

Our method closes the gap in all instances within time limit



Vertical axis: The number of instances solved to optimality

Reference: **Jiang, N.**, Xie, W. (2024). The Terminator: An Integration of Inner and Outer Approximations for Solving Wasserstein Distributionally Robust Chance Constrained Programs via Variable Fixing. Forthcoming at *INFORMS Journal on Computing.*