Copositive Duality for Discrete Energy Markets

Cheng Guo

School of Mathematical and Statistical Sciences
Clemson University

Joint work with Merve Bodur and Josh Taylor

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Design A Pricing Scheme for Energy Markets with Discreteness

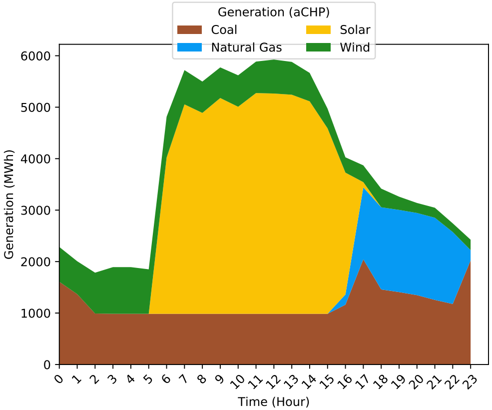
- Pricing is central to energy markets
- Electricity prices are based on shadow prices
  - Idealized market structure
- Discrete decisions in day-ahead market: start-up, on/off statuses
- Our solution: convexification of MIP

Outline

- Convexification of Unit Commitment using copositive programming
- Pricing Scheme in Discrete Energy Markets
  - Pricing and individual rationality in spot market
  - Pricing and individual rationality in day-ahead market
- Cutting plane algorithm for copositive programs

Unit Commitment (UC) Problem

- In the day-ahead market, decide the operation schedule of generators at each hour



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MIP Model for Unit Commitment

$$
\begin{aligned}
\min \quad & \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \left(c_g^p p_{gt} + c_g^u u_{gt} \right) & \\
\text{s.t.} \quad & \sum_{g \in \mathcal{G}} p_{gt} = d_t & \forall t \in \mathcal{T} \\
& \mathbf{a}_{jgt}^{\phi\top} \mathbf{x} = b_{jgt} & \forall j = 1, ..., m, g \in \mathcal{G}, t \in \mathcal{T} \\
& \mathbf{x} \in \mathbb{R}_+^n & \\
& z_{gt} \in \{0, 1\} & \forall g \in \mathcal{G}, t \in \mathcal{T}
\end{aligned}
$$

- p_{gt}: production level
- u_{gt}: turn on decision
- z_{gt}: on/off status
- $\ddot{\phi}$: slack variables
- $\mathbf{x}^\top = (\mathbf{z}^\top, \mathbf{u}^\top, \mathbf{p}^\top, \ddot{\phi}^\top)$

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MIP → Completely Positive Programming (CPP) (Burer, 2009)

$\mathcal{P}^{\text{MIP}}$ (nonconvex):

$$\begin{aligned} \min \quad & \mathbf{c}^\top \mathbf{x} \\ \text{s.t.} \quad & a_j^\top \mathbf{x} = b_j, && \forall j = 1, \dots, m \\ & x^k \in \{0,1\}, && \forall k \in \mathcal{B} \\ & \mathbf{x} \in \mathbb{R}^n_+ \end{aligned}$$

$\mathcal{P}^{\text{CPP}}$ (convex):

$$\begin{aligned} \min \quad & \mathbf{c}^\top \mathbf{x} \\ \text{s.t.} \quad & \mathbf{a}_j^\top \mathbf{x} = b_j && \forall j = 1, \dots, m \\ & \mathbf{a}_j^\top X \mathbf{a}_j = b_j^2 && \forall j = 1, \dots, m \\ & x^k = X_{kk} && \forall k \in \mathcal{B} \\ & \begin{bmatrix} 1 & \mathbf{x}^\top \\ \mathbf{x} & X \end{bmatrix} \in \mathcal{C}^* \end{aligned}$$

- If $x^k \in \{0,1\}$, then $x^k = (x^k)^2$
- Let $X = \mathbf{x}\mathbf{x}^\top$, Enforce $x^k = X_{kk}$
- Constraints to enforce $X = \mathbf{x}\mathbf{x}^\top \rightarrow$ *there are different ways to do this for MIQP!*
  - Reformulation-Linearization Technique (RLT) constraint: $\mathbf{a}_j^\top X \mathbf{a}_j = b_j^2$
  - $\begin{bmatrix} 1 & \mathbf{x}^\top \\ \mathbf{x} & X \end{bmatrix} \in \mathcal{C}^*$
- Strong duality holds for CPP under regularity condition is satisfied.

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Setup of the Energy Market

- Supply: power plants, demand: utilities
- Independent system operator (ISO) holds auctions to match supply and demand
  - Day-ahead market: unit commitment
  - Spot market: no discrete decision

Pricing Scheme in Spot Market

- Spot market: ISO minimizes total cost

$$
\begin{aligned}
\min_{p_{gt}} \quad & \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} c_g^p p_{gt} \\
\text{s.t.} \quad & \sum_{g \in \mathcal{G}} p_{gt} = d_t, \quad \forall t \in \mathcal{T} \qquad\qquad (\lambda_t) \\
& (p_{gt}) \in X'_{gt}, \quad \forall g \in \mathcal{G}, t \in \mathcal{T}
\end{aligned}
$$

- Let the optimal primal and dual solution be p^*_{gt} and λ^*_t.
- λ^*_t is the electricity price: More demand $\rightarrow$ more expensive technology $\rightarrow$ higher λ^*_t

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λ_t^* Guarantees Individual Rationality in Spot Markets

- Profit-maximizing problem for g has the same solution as the ISO's problem:

$$\max_{p_{gt}} \sum_{t \in \mathcal{T}} (\lambda_t^* - c_g^p) p_{gt}$$
$$\text{s.t.} \quad (p_{gt}) \in X'_{gt}, \quad \forall t \in \mathcal{T}$$

- How to prove this? Decompose the Lagrangified ISO's problem

Proof for Individual Rationality in Spot Markets

- Lagrangify the demand constraint in the min-cost problem using λ_t^*. Due to convexity, p_{gt}^* is optimal to the following:

$$\begin{aligned} \min_{p_{gt}} \quad & \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}} c_g^p p_{gt} + \sum_{t\in\mathcal{T}} \lambda_t^* (\sum_{g\in\mathcal{G}} p_{gt} - d_t) \\ \text{s.t.} \quad & (p_{gt}) \in X'_{gt}, \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \end{aligned}$$

- Drop constant term $\lambda_t^* d_t$, reverse the sense:

$$\begin{aligned} \max_{p_{gt}} \quad & \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}} (\lambda_t^* - c_g^p) p_{gt} \\ \text{s.t.} \quad & (p_{gt}) \in X'_{gt}, \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \end{aligned}$$

- Decomposable by g

Pricing for Markets with Discrete Decisions is Challenging

- No dual price in MIP
- Literature on discrete energy market
  - Restricted pricing [O'Neil et al., 2005]
  - Convex hull pricing [Hogan and Ring, 2003; Gribik et al., 2007]
    Extended locational marginal pricing
- Literature on indivisible goods
  - Discrete convexity [Danilov et al., 2001; Baldwin and Klemperer, 2019]
  - Alpha-price mechanism [Milgrom and Watt, 2022]
- Still an open question

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Recap: Unit Commitment Problem & CPP Reformulation

$$\begin{aligned}
\mathcal{UC} : \min \quad & \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}} \left(c_g^p p_{gt} + c_g^u u_{gt}\right) & & \\
\text{s.t.} \quad & \sum_{g\in\mathcal{G}} p_{gt} = d_t & \forall t \in \mathcal{T} & \quad (\lambda_t) \\
& \mathbf{a}_{jgt}^{\phi\top}\mathbf{x} = b_{jgt} & \forall j = 1,\ldots,m, g\in\mathcal{G}, t\in\mathcal{T} & \\
& z_{gt} \in \{0,1\} & \forall g\in\mathcal{G}, t\in\mathcal{T} &
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}^{\text{CPP}} : \min \quad & \mathbf{c}^\top\mathbf{x} & \\
\text{s.t.} \quad & \mathbf{a}_j^\top\mathbf{x} = b_j & \forall j = 1,\ldots,m \\
& \mathbf{a}_j^\top X\mathbf{a}_j = b_j^2 & \forall j = 1,\ldots,m \\
& x^k = X_{kk} & \forall k \in \mathcal{B} \\
& \begin{bmatrix} 1 & \mathbf{x}^\top \\ \mathbf{x} & X \end{bmatrix} \in \mathcal{C}^* &
\end{aligned}$$

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Convexification of UC

- CPP reformulation:

$$
\begin{aligned}
\mathcal{UC}^{\text{CPP}} = \min \quad & \sum_{g\in\mathcal{G}} \sum_{t\in\mathcal{T}} \left(c_g^p p_{gt} + c_g^u u_{gt} \right) & & \\
\text{s.t.} \quad & \sum_{g\in\mathcal{G}} p_{gt} = d_t & \forall t \in \mathcal{T} & \quad (\lambda_t) \\
& \mathbf{a}_{jgt}^{\phi\top} \mathbf{x} = b_{jgt} & \forall j = 1,\dots,m, g \in \mathcal{G}, t \in \mathcal{T} & \quad (\phi_{jgt}) \\
& \text{Tr}(\mathbf{a}_t^{\lambda} \mathbf{a}_t^{\lambda\top} X) = d_t^2 & \forall t \in \mathcal{T} & \quad (\Lambda_t) \\
& \text{Tr}(\mathbf{a}_{jgt}^{\phi} \mathbf{a}_{jgt}^{\phi\top} X) = b_{jgt}^2 & \forall j = 1,\dots,m, g \in \mathcal{G}, t \in \mathcal{T} & \quad (\Phi_{jgt}) \\
& z_{gt} = Z_{gt} & \forall g \in \mathcal{G}, t \in \mathcal{T} & \quad (\delta_{gt}) \\
& \begin{bmatrix} 1 & x^\top \\ x & X \end{bmatrix} \in \mathcal{C}_{n+1}^* & & \quad (\Omega)
\end{aligned}
$$

- Dual problem:

$$
\begin{aligned}
\mathcal{UC}^{\text{COP}} = \max \quad & \sum_{t\in\mathcal{T}} \left(d_t \lambda_t + d_t^2 \Lambda_t + \sum_{j=1}^{m} \sum_{g\in\mathcal{G}} \left(b_{jgt} \phi_{jgt} + b_{jgt}^2 \Phi_{jgt} \right) \right) \\
\text{s.t.} \quad & (\boldsymbol{\lambda}, \boldsymbol{\phi}, \boldsymbol{\Lambda}, \boldsymbol{\Phi}, \boldsymbol{\delta}, \Omega) \in \mathcal{F}^{\text{COP}}
\end{aligned}
$$

Shadow Price for Day-Ahead Market: Copositive Dual Pricing (CDP)

Let $(\boldsymbol{\lambda}^*, \boldsymbol{\phi}^*, \boldsymbol{\Lambda}^*, \boldsymbol{\phi}^*)$ be an optimal solution for $\mathcal{UC}^{\text{COP}}$. Under the CDP mechanism, at hour t the system operator:

(i) collects from the load:

$$d_t \lambda_t^* + d_t^2 \Lambda_t^* + \sum_{g \in \mathcal{G}} \sum_{j=1}^{m} \left(b_{jgt} \phi_{jgt}^* + b_{jgt}^2 \Phi_{jgt}^* \right)$$

(ii) pays to the generator g:

$$p_{gt}^* \lambda_t^* + P_{gt}^* \Lambda_t^* + \sum_{j=1}^{m} \left(\mathbf{a}_{jgt}^{\phi} \mathbf{x}^* \phi_{jgt}^* + \text{Tr}(\mathbf{a}_{jgt}^{\phi} \mathbf{a}_{jgt}^{\phi \top} X^*) \Phi_{jgt}^* \right) + \sum_{g' \in \mathcal{G} \setminus \{g\}} f(\Lambda_t^*, p_{gt}^*, p_{g't}^*)$$

Proof for Individual Rationality in Day-Ahead Markets

- Lagrangified CPP:

$$
\begin{aligned}
\min \quad & \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\left(c_g^p p_{gt} + c_g^u u_{gt}\right) + \lambda_t^* \sum_{t\in\mathcal{T}}(d_t - \sum_{g\in\mathcal{G}} p_{gt}) + \Lambda_t^* \sum_{t\in\mathcal{T}}(d_t^2 - \text{Tr}(\mathbf{a}_t^\lambda \mathbf{a}_t^{\lambda\top} X)) \\
\text{s.t.} \quad & \mathbf{a}_{jgt}^{\phi\top}\mathbf{x} = b_{jgt} && \forall j = 1,\dots,m, g\in\mathcal{G}, t\in\mathcal{T} \\
& \text{Tr}(\mathbf{a}_{jgt}^{\phi}\mathbf{a}_{jgt}^{\phi\top} X) = b_{jgt}^2 && \forall j = 1,\dots,m, g\in\mathcal{G}, t\in\mathcal{T} \\
& z_{gt} = Z_{gt} && \forall g\in\mathcal{G}, t\in\mathcal{T} \\
& \begin{bmatrix} 1 & x^\top \\ x & X \end{bmatrix} \in \mathcal{C}_{n+1}^*
\end{aligned}
$$

- Idea: decompose this by g. But how?
  - First idea: make the conic constraint decomposable
  - Second idea: make $\Lambda_t^* = 0$
- A decomposable "Lagrangified MIP"

$$
\begin{aligned}
\min \quad & \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\left(c_g^p p_{gt} + c_g^u u_{gt}\right) + \lambda_t^* \sum_{t\in\mathcal{T}}(d_t - \sum_{g\in\mathcal{G}} p_{gt}) \\
\text{s.t.} \quad & \mathbf{a}_{jgt}^{\phi\top}\mathbf{x} = b_{jgt} && \forall j = 1,\dots,m, g\in\mathcal{G}, t\in\mathcal{T}
\end{aligned}
$$

Some Other Analytical Results

- System operators: Revenue from load = Payment to generators
- Generators: Total revenue = total costs (revenue neutrality)
- Supports market equilibrium
- A modified version of CDP that ensures individual revenue adequacy and uses linear prices
  - Results for CDP can be extended to this

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Solve the Dual Pricing Problem (A Copositive Program)

$$\mathcal{UC}^{\text{COP}} = \max \sum_{t\in\mathcal{T}} \Big(d_t\lambda_t + d_t^2\Lambda_t + \sum_{j=1}^{m}\sum_{g\in\mathcal{G}} \big(b_{jgt}\phi_{jgt} + b_{jgt}^2\Phi_{jgt}\big) \Big)$$
$$\text{s.t.} \quad (\boldsymbol{\lambda}, \boldsymbol{\phi}, \boldsymbol{\Lambda}, \boldsymbol{\Phi}, \boldsymbol{\delta}, \Omega) \in \mathcal{F}^{\text{COP}}$$

- $\mathcal{F}^{\text{COP}}$ includes conic constraint $\Omega \in \mathcal{C}_{n+1}$
- In literature: solved with SDP restriction
  - Define $\mathcal{S}^+$ and $\mathcal{N}$ ($\ni X_{ij} \geq 0, \forall i, j$)
  - $\mathcal{S}^+ + \mathcal{N} \subseteq \mathcal{C}$

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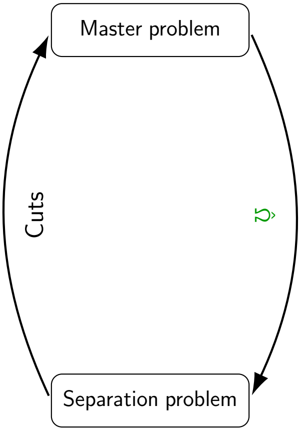
A Novel Cutting Plane Algorithm for Solving COP Exactly

$$\begin{aligned} \max_{\Omega,\boldsymbol{\lambda}} \quad & \mathbf{q}^\top \boldsymbol{\lambda} + \mathrm{Tr}(H^\top \Omega) \\ \text{s.t.} \quad & \mathbf{d}^\top \boldsymbol{\lambda} + \mathrm{Tr}(D_i^\top \Omega) = g_i, \quad \forall i = 1, \ldots, m \\ & \boldsymbol{\lambda} \geq \mathbf{0} \\ & \Omega \in \mathcal{C}^{n_c} \end{aligned}$$

- Separation problem [Anstreicher, 2020]:

$$\begin{aligned} \max_{w,\boldsymbol{u},\boldsymbol{z}} \quad & w \\ \text{s.t.} \quad & \hat{\Omega}\mathbf{z} \leq -w\mathbf{1} + M(1 - \mathbf{u}) \\ & \mathbf{1}^\top \mathbf{u} \geq q \\ & w \geq 0 \\ & 0 \leq \mathbf{z} \leq \mathbf{u} \\ & \mathbf{u} \in \{0, 1\}^{n_c} \end{aligned}$$

- If $\bar{w} > 0$, add the cut: $\bar{\mathbf{z}}^\top \Omega \bar{\mathbf{z}} \geq 0$



Tighten the Master Problem Via Second-Order Cone Program

$$
\begin{aligned}
\max_{\Omega,\boldsymbol{\lambda}} \quad & \mathbf{q}^\top \boldsymbol{\lambda} + \mathrm{Tr}(H^\top \Omega) \\
\text{s.t.} \quad & \mathbf{d}^\top \boldsymbol{\lambda} + \mathrm{Tr}(D_i^\top \Omega) = g_i, \quad \forall i = 1, ..., m \\
& \boldsymbol{\lambda} \geq \mathbf{0} \\
& V + N = \Omega \\
& N \geq 0 \\
& V \in \mathcal{S}_n^+ \\
& V_{ii} \geq 0 \quad \forall i = 1, \cdots, n \\
& V_{ii} V_{jj} \geq V_{ij}^2 \quad \forall i \neq j; i, j = 1, \cdots, n \\
& \Omega \in \mathcal{C}^{n_c}
\end{aligned}
$$

- Converges to a feasible (not necessarily optimal) solution
- No worse than the SDP approximation $\mathcal{S}^+ + \mathcal{N} \subseteq \mathcal{C}$

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Comments and Performance of Cutting Plane Algorithms

- Straightforward to implement
  (vs simplicial partition [Bundfuss and Dür, 2008])
- Experiment on the max clique problem (2nd DIMACS dataset)
  - Cutting plane is more accurate and sometimes faster than the SDP approximation
- Significant speedup with the SOC-strengthened master problem
- To be improved:
  - Speed up the separation problem
  - Bounding the master problem at initialization
  - Tighter master problem
  - Other types of cuts

Summary

- A notion of duality for discrete problems
- Pricing scheme for discrete energy markets with good properties
- Novel cutting plane algorithm for copositive programs

Future Research

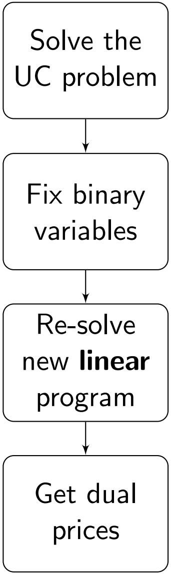
- Optimization and duality theory play important roles in classical economic models: utility theory, pricing, game theory, $\cdots$
- Real-life markets (e.g. energy markets) are not always ideal: discreteness, nonlinearities, uncertainties, market power, $\cdots$
- More realistic optimization models and more rigorous analysis are needed for energy markets and other economic problems

Restricted Pricing (RP)

- Used in many ISOs such as PJM, ISO-NE, CAISO, and ERCOT

$$
\begin{aligned}
\min_{u_g, p_g} \quad & \sum_{g \in \mathcal{G}} f(u_g, p_g) \\
\text{s.t.} \quad & \sum_{g \in \mathcal{G}} p_g = d \qquad\qquad (\lambda^{\mathsf{RP}}) \\
& (u_g, p_g) \in X_g, \quad \forall g \in \mathcal{G} \\
& u_g = u_g^*
\end{aligned}
$$

- Generators are not necessarily profitable



Convex Hull Pricing (CHP)

- Less profit deficient than RP
- Value function $v(d)$ is parameterized by the demand d

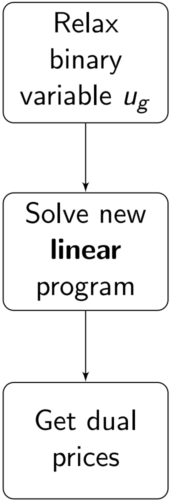
$$
\begin{aligned}
v(d) = \min_{u_g, p_g} \quad & \sum_{g \in \mathcal{G}} f(u_g, p_g) \\
\text{s.t.} \quad & \sum_{g \in \mathcal{G}} p_g = d \qquad\qquad (\lambda^{\mathsf{CHP}}) \\
& (u_g, p_g) \in X_g, \quad \forall g \in \mathcal{G}
\end{aligned}
$$

- Take subgradient as the price:

Approximated Convex Hull Pricing (aCHP)

- Implemented by MISO
- Should use a tight UC formulation

$$
\begin{aligned}
\min_{u_g, p_g} \quad & \sum_{g \in \mathcal{G}} f(u_g, p_g) \\
\text{s.t.} \quad & \sum_{g \in \mathcal{G}} p_g = d \qquad\qquad (\lambda^{\mathrm{aCHP}}) \\
& (u_g, p_g) \in X_g, \quad \forall g \in \mathcal{G} \tilde{X}_g, \quad \forall g \in \mathcal{G}
\end{aligned}
$$



Primal-dual Pricing

- Seeks a revenue-adequate price

$$\begin{aligned} \min \quad & \text{Duality gap} \\ \text{s.t.} \quad & \text{Primal LP constraints} \\ & \text{Dual LP constraints} \\ & \text{Integrality restrictions} \\ & \text{Revenue-adequacy constraints} \end{aligned}$$

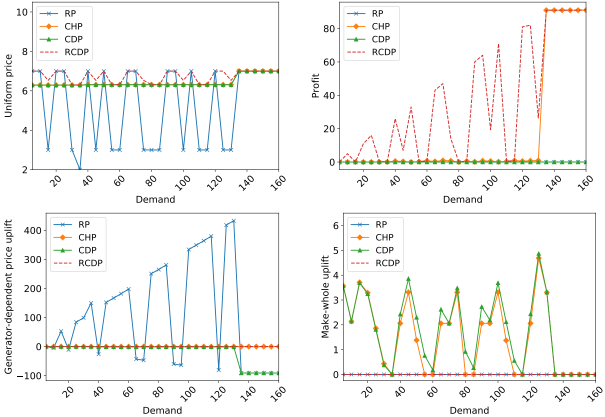
Experiments: COP Algorithm Comparison

- Max clique problem for testing COP algorithms
  - Cutting plane usually converges in a few iterations and sometimes faster than the SDP approximation

				Mosek			Cutting plane	
Instance	$\lvert\mathcal{N}\rvert$	$\lvert\mathcal{E}\rvert$	ω	Obj	Gap(%)	Time(sec)	Time(sec)	#Iter
c-fat200-1	200	1534	12	12	0	566.81	13.87	2
c-fat200-2	200	3235	24	24	0	638.72	18.90	2
c-fat200-5	200	8473	58	60.35	3.89	606.33	12.19	2
hamming6-2	64	1824	32	32	0	1.51	6.05	2
hamming6-4	64	704	4	4	0	1.59	1.55	4
johnson8-2-4	28	210	4	4	0	0.20	9.53	2
johnson8-4-4	70	1855	14	14	0	2.47	11.82	2
johnson16-2-4	120	5460	8	8	0	31.88	62.75	2
keller4	171	9435	11	13.47	18.34	426.16	-	-
MANN_a9	45	918	16	17.48	8.47	0.45	547.62	2

Experiments: Scarf's Example

- Although CDP has generator-dependent payment, its behavior is more similar to CHP than RP
- RCDP requires no uplift payment



Experiments: Performance of UC Instances I

- Strengthened cutting plane algorithm is much faster
- Negative optimality gaps:
  - No strong duality
  - Bounds on Ω
  - SOC constraints
  - Revenue adequacy constraints (for RCDP)

Table: Time (seconds), Optimality Gap (%) and Number of Iterations of Cases 1-3

	CDP LP			CDP SOC			RCDP LP			RCDP SOC		
Case	Time	Gap	#Iter	Time	Gap	#Iter	Time	Gap	#Iter	Time	Gap	#Iter
1	187.4	0	1051	80.2	-0.34	121	244.5	0	1303	50.4	0	55
2	195.3	0	1038	89.1	0	62	249.5	0	1339	76.2	0	55
3	536.7	-4.97	1635	68.5	-7.28	131	319.5	-4.18	1310	78.0	-7.34	146

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Experiments: Performance of UC Instances II

- Performance is more likely to be affected by the number of generators than the length of the time horizon
- Separation problem takes much longer time than the master problem

Table: Optimality Gap (%) and Number of Iterations of Cases 4-10

	CDP LP		CDP SOC		RCDP LP		RCDP SOC	
Case	Gap	#Iter	Gap	#Iter	Gap	#Iter	Gap	#Iter
4	14.41	3279	3.32	212	15.43	3212	3.77	207
5	14.80	3750	5.85	230	15.03	2985	4.15	243
6	9.08	2979	-1.25	310	4.95	3247	-0.93	394
7	88.78	1171	18.71	99	110.13	909	17.63	109
8	49.04	3895	16.13	228	27.06	5496	0.91	344
9	106.28	1549	21.35	102	130.37	1734	21.52	132
10	80.94	1724	17.96	224	132.40	1826	18.12	250