

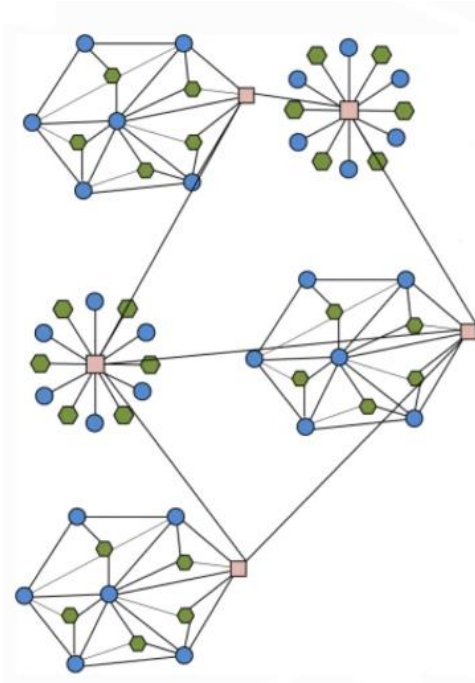
Network Design Tools & Multicommodity Flows

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- Collaborations with Dan Bienstock
- A simplified multicommodity flow model
- Solution outline
- Observations
- Numerical confirmation
- Epilogue

Network Design Tools -- Our (early 1990s) Little Cottage Industry at Bellcore



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Data: Square nodes have large fixed cost (switches), Hexagonal nodes have medium fixed cost, Circular nodes and links have per unit cost. There is partial capacity.

Problem: Given end-to-end new demands, determine min-cost expansion to accommodate all new demands.

Type: MILP

Technologies: SONET, Frame Relay, ATM, IP over optics.

Novel Extension -- Beyond Typical Regulated Monopoly-Inspired Models

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Combined Network Design and Multiperiod Pricing: Modeling, Solution Techniques, and Computation

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In this paper we describe an efficient algorithm for solving novel optimization models arising in the context of multiperiod capacity expansion of optical networks. We assume that the network operator must make investment decisions over a multiperiod planning horizon while facing rapid changes in transmission technology, as evidenced by a steadily decreasing per-unit cost of capacity. We derive from traditional and nontraditional models in which demands are given as input parameters, and the objective is to minimize capacity deployment costs. Instead, we assume that the carrier sets end-to-end prices of bandwidth at each period of the planning horizon. These prices determine the demands that are to be met, using a plausible and explicit price-demand relationship; the resulting demands must then be met, requiring an investment in capacity. The objective of the optimization is now to simultaneously select end-to-end prices of bandwidth and network capacity at each period of the planning horizon, so as to maximize the overall net present value of expanding and operating the network. In the case of typical large-scale optical networks with protection requirements, the resulting optimization problems pose significant challenges to standard optimization techniques. The complexity of the model, its nonlinear nature, and the large size of realistic problem instances motivate the development of efficient and scalable solution techniques. We show that while general purpose nonlinear solvers are typically not adequate for the task, a specialized decomposition scheme is able to handle large-scale instances of this problem in reasonable time, producing solutions whose net present value is within a small tolerance of the optimum.

Subject classification: communications; facility/location planning; capacity expansion; design; manufacturing/procurement; programming; nonlinear.

Area of review: Telecommunications.

History: Received May 2003; revisions received December 2003, January 2005; accepted January 2005.

1. Introduction

In this paper, we describe a novel model and develop efficient algorithms for addressing capacity expansion and allocation combined with bandwidth pricing in the context of designing resilient optical transport networks.

A network is modeled as a set of nodes, representing cities and/or metropolitan areas, which are connected by links representing physical routes owned by the network operator; transmission systems deployed on links are used to carry traffic. The carrier, or network operator, makes investments for deploying these systems and incurs periodic operating expenses, while collecting revenue from carrying demand for customers.

We consider network planning over a time horizon that spans many years. During such a period several generations of transmission technologies will typically emerge. Although a newer system may have a higher deployment and operating cost, the magnitude of capacity improvement

can be expected to far outweigh that of cost increase. As a result, a new technology results in a lower cost per unit of capacity than previous ones, thus making it an attractive candidate for new deployment.

In addition, after a new technology becomes available, its deployment cost decreases over time due to a learning effect. As a result, there may be an economic incentive to delay the deployment of a new technology until it yields savings from future cost reductions.

Such a dynamic technology environment immediately poses two interesting problems. First, timing: When should the operator start to deploy the new systems and phase out old technologies? Second, sizing: How much capacity should be deployed on each link at each time period?

Traditional network-planning problems have been discussed extensively in the literature, and it would be impractical to provide a thorough review here. See Alleva et al. (1998), Bakkerstein et al. (1995), Bienstock et al.

Data: Demand-price elasticity and previous fixed costs

Problem: maximize profit (revenue – cost) over multi-year planning horizon

Type: Novel (very) non-linear objective function

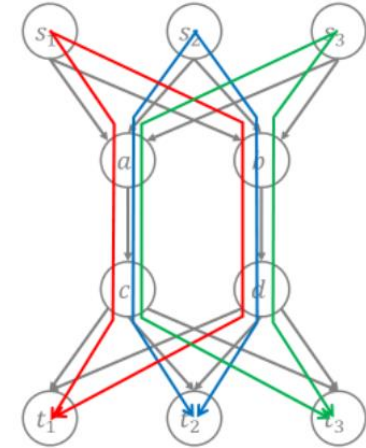
Context: Inspired by companies like Level3 who were putting tons of dark fibers to own telecom infrastructure globally (early 2000s)

Multicommodity Flow Problem (myMCF) -- Can one solve it 'analytically'?*

- Include fixed costs? Too complex
- Path-based solution? Maybe later
- Cost minimization? Start with capacity-guided:

Given an arbitrary graph G with weighted symmetric adjacency matrix $A = A^T = (a_{ij})$ and traffic matrix $T = (t_{kl})$, the required flow from k to l , route of t_{kl} at k and each intermediate node j is proportional to a_{ij} until flow reaches k .

Trivial bookkeeping exercise?



*Joint work with O. Narayan, UCSC.

Extension to non-uniform edge weights and arbitrary demand

$G = (N, L)$ is a connected graph

A = an $N \times N$ symmetric matrix of non-negative weighted adjacencies

$d_j = \sum_i a_{ij}$ is weighted degree of node j

D = an $N \times N$ diagonal matrix of weighted adjacencies

$L = D - A$ is the graph Laplacian

$\Pi = (N^{-1}) \mathbf{1}_{N \times N}$ perturbation matrix with N^{-1} in all entries

$M = L + \Pi$ is the perturbed graph Laplacian

$T = (t_{kl})$ is an $N \times N$ symmetric non-negative flow matrix

$T_k = \sum_l t_{kl}$ the total flow out of node k

$\tilde{T} = (T_k)$ is an $N \times N$ diagonal matrix of total nodal out flows

$L_T = \tilde{T} - T$ for symmetric T

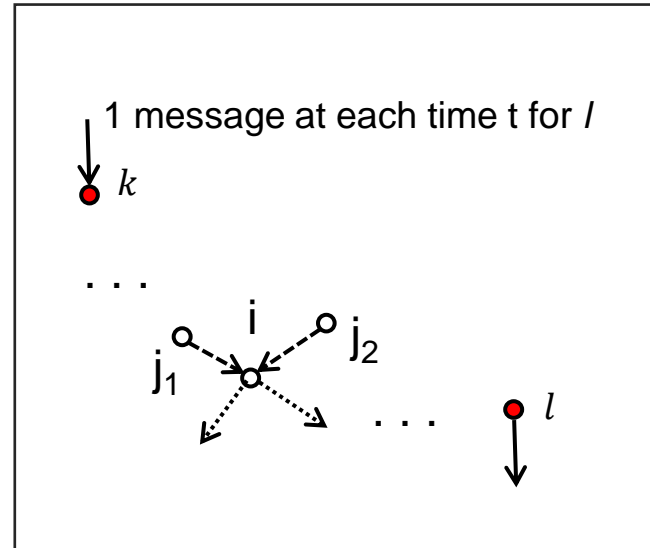
Load on each node in myMCF for a specific commodity k-l

- Start **with 1 unit of flow** for a given source-destination pair (k, l)
- Let f_i^{kl} be the flow at node i . Then

$$f_i^{kl} = \delta_{ik} - \delta_{il} + \sum_{j \sim i} \frac{a_{ji}}{d_j} f_j^{kl}$$

with the boundary condition $f_l^{kl} = 0^*$. In vector form

$$\mathbf{f}^{kl} = \mathbf{v}^k - \mathbf{v}^l + \mathbf{AD}^{-1} \mathbf{f}^{kl} \quad (*)$$



*We'll add back the total flow terminating at each l in (*****).

Derivation of random load, 1

$$\mathbf{f}^{kl} = \mathbf{v}^k - \mathbf{v}^l + \mathbf{A}\mathbf{D}^{-1}\mathbf{f}^{kl} \text{ with } f_i^{kl} = 0 \quad (*)$$

Using the substitution $r_j^{kl} = f_j^{kl}/d_j$, or $\mathbf{r}^{kl} = \mathbf{D}^{-1}\mathbf{f}^{kl}$ (*) becomes

$$\mathbf{D}\mathbf{r}^{kl} = \mathbf{v}^k - \mathbf{v}^l + \mathbf{A}\mathbf{r}^{kl}$$

Or

$$\mathbf{L}\mathbf{r}^{kl} = \mathbf{v}^k - \mathbf{v}^l \quad (**)$$

If \mathbf{L} were invertible we could just write $\mathbf{r}^{kl} = \mathbf{L}^{-1}(\mathbf{v}^k - \mathbf{v}^l)$ and we'd be nearly there. Need some adjustment. Notice that (**) is already telling us we have an under-determined system of equations. We add a homogeneous boundary condition $r_l^{kl} = 0$ which we'll update at the end.

Aside – Uniform Perturbation of the Laplacian

Let $\{0 = \lambda_0 \leq \lambda_1 \leq \dots \leq \lambda_{N-1} \leq 2d_{\max}\}$ be the eigenvalues and $\{\xi^\alpha; \alpha = 0, 1, \dots, N-1\}$ be the corresponding orthonormal eigenvectors of \mathbf{L} . Recall corresponding to $0 = \lambda_0$, have

$$(\xi^0)^T = (1, 1, \dots, 1)^T / \sqrt{N}$$

Consider projection $\mathbf{\Pi} = \left(\frac{1}{N}\right)_{N \times N}$ and $\mathbf{M} = \mathbf{L} + \mathbf{\Pi}$. Notice that \mathbf{M} has the same eigen system as \mathbf{L} except that $\lambda_0(\mathbf{M}) = 1$.

$$\begin{aligned} \mathbf{M} \xi^0 &= (\mathbf{L} + \mathbf{\Pi}) \xi^0 = 0 + \sum_i \xi_i^0 \mathbf{1}_N = N \left(\frac{1}{N}\right) \left(\frac{1}{\sqrt{N}}\right) \mathbf{1}_N = \xi^0 \\ \text{and} \\ \mathbf{M} \xi^\alpha &= (\mathbf{L} + \mathbf{\Pi}) \xi^\alpha = \mathbf{L} \xi^\alpha + \mathbf{\Pi} \xi^\alpha = \lambda_\alpha \xi^\alpha \text{ with } \alpha > 0. \end{aligned}$$

Above means \mathbf{M} has a simple tractable inverse in terms of λ s and ξ s. Also observe that

$$\mathbf{M} = \mathbf{L} + \mathbf{\Pi} \rightarrow \mathbf{M}\mathbf{\Pi} = \mathbf{L}\mathbf{\Pi} + \mathbf{\Pi}^2 = \mathbf{\Pi}^2 = \mathbf{\Pi} \rightarrow \mathbf{M}^{-1}\mathbf{\Pi} = \mathbf{\Pi} \quad (***)$$

Derivation of random load, 2

Let us return to $\mathbf{L} \mathbf{r}^{kl} = \mathbf{v}^k - \mathbf{v}^l$ with $r_l^{kl} = 0$ (**).

Adding $\mathbf{\Pi} \mathbf{r}^{kl}$ to both sides of (**), we get

$$\mathbf{L} \mathbf{r}^{kl} + \mathbf{\Pi} \mathbf{r}^{kl} = (\mathbf{v}^k - \mathbf{v}^l) + \mathbf{\Pi} \mathbf{r}^{kl}$$

$$\mathbf{M} \mathbf{r}^{kl} = (\mathbf{v}^k - \mathbf{v}^l) + \mathbf{\Pi} \mathbf{r}^{kl}$$

So at last

$$\begin{aligned} \mathbf{r}^{kl} &= \mathbf{M}^{-1}[(\mathbf{v}^k - \mathbf{v}^l) + \mathbf{\Pi} \mathbf{r}^{kl}] = \mathbf{M}^{-1}(\mathbf{v}^k - \mathbf{v}^l) + \mathbf{M}^{-1}\mathbf{\Pi} \mathbf{r}^{kl} \\ &= \mathbf{M}^{-1}(\mathbf{v}^k - \mathbf{v}^l) + \mathbf{\Pi} \mathbf{r}^{kl} \quad (\text{by (***)}) \\ &= \mathbf{M}^{-1}(\mathbf{v}^k - \mathbf{v}^l) + \theta(kl)\mathbf{1}_N \end{aligned}$$

And we can get $\theta(kl)$ from the boundary condition.

Derivation of random load, 3

$$0 = r_l^{kl} = [\mathbf{M}^{-1}(\mathbf{v}^k - \mathbf{v}^l)]_l + \theta(kl)$$

$$0 = [\mathbf{M}^{-1}\mathbf{v}^k]_l - [\mathbf{M}^{-1}\mathbf{v}^l]_l + \theta(kl)$$

$$0 = [\mathbf{M}^{-1}]_{lk} - [\mathbf{M}^{-1}]_{ll} + \theta(kl)$$

$$\theta(kl) = [\mathbf{M}^{-1}]_{ll} - [\mathbf{M}^{-1}]_{lk}$$

Now going back to r_j^{kl}

$$r_j^{kl} = [\mathbf{M}^{-1}]_{jk} - [\mathbf{M}^{-1}]_{jl} - [\mathbf{M}^{-1}]_{lk} + [\mathbf{M}^{-1}]_{ll} \quad (****)$$

We can now get f_j^{kl} for t_{kl} units from (****)

Derivation of random load, 4 – Final Result

So flow f_j^{kl} for t_{kl} units of flow is just $t_{kl} r_j^{kl} d_j$.

If Λ_j denotes the total flow at node j for *all* commodities, then

$$\begin{aligned}\Lambda_j &= \sum_k \sum_l t_{kl} f_j^{kl} + \sum_k t_{kj} \\ &= d_j \sum_k \sum_l t_{kl} (r_j^{kl} = [\mathbf{M}^{-1}]_{jk} - [\mathbf{M}^{-1}]_{jl} - [\mathbf{M}^{-1}]_{lk} + [\mathbf{M}^{-1}]_{ll}) + T_j\end{aligned}$$

And at last, if the demand matrix is symmetric ($t_{kl})=(t_{lk})$

$$\Lambda_j = d_j \text{Tr}[\mathbf{L}_T \mathbf{M}^{-1}] + T_j$$

$$\mathbf{\Lambda} = \mathbf{D} \text{Tr}[\mathbf{L}_T \mathbf{M}^{-1}] + \tilde{\mathbf{T}} \cdot \mathbf{1}_N$$

(*****)

Derivation of random load, 5 – Case of All-to-All Uniform Flows

If $t_{kl} = 1$ for all $k \neq l$ then

$$\Lambda_j = d_j N \sum_{a>0} \frac{1}{\lambda_a} + (N - 1)$$

This is seen more easily from re-writing (*****) in this form

$$\Lambda_j = d_j \sum_k \sum_l t_{kl} \sum_{\alpha>0} \frac{1}{\lambda_\alpha} \xi_l^\alpha (\xi_l^\alpha - \xi_k^\alpha) + T_j$$

Load due to Net MCF

In derivation of

$$\Lambda_j = (N - 1) + (N \sum_{a \geq 1} \frac{1}{\lambda_a}) d_j$$

we allowed RW to go through a transit node multiple times, each time contributing to load. We may define an alternative load, $\bar{\Lambda}_j$, which measures the *net flow* through node.

Thus for neighbors i & j net inflow to node i from neighbor j

$$= \frac{p_j^{kl}}{d_j} - \frac{p_i^{kl}}{d_i}$$

Thus in equilibrium

$$\bar{\Lambda}_j^{kl} = \frac{1}{2} \sum_{i \sim j} |r_i^{kl} - r_j^{kl}| + \frac{1}{2} (\delta_{ik} + \delta_{il})$$

Adding up for all (k, l)

$$\bar{\Lambda}_j = \frac{1}{2N(N-1)} \sum_{k \neq l} \sum_{i \sim j} |r_i^{kl} - r_j^{kl}| + \frac{1}{N}$$

Load due to Net MCF

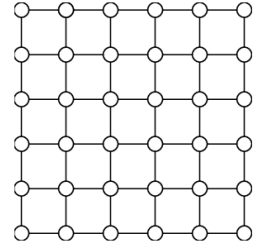
After substituting for r_j^{kl} from (3),

$$\bar{\Lambda}_j = \frac{1}{2N(N-1)} \sum_{k \neq l} \sum_{i \sim j} \sum_{a > 0} |(\xi_k^a - \xi_l^a) \frac{1}{\lambda_a} (\xi_j^a - \xi_i^a)| + \frac{1}{N}$$

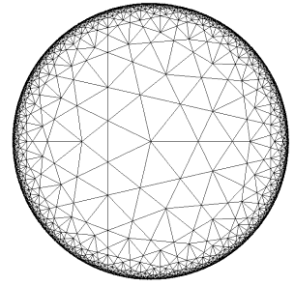
where as before $\{\lambda_a, \xi^a\} \in \text{eigSys}(\mathbf{L})$.

Maximum Load Λ_d for Some Prototypical Graphs Using Shortest Paths

Square Lattice $\Lambda_d \sim O(N^{\frac{3}{2}})$ for large N



Hyperbolic grids $\Lambda_d \sim N^2$ for large N



For random graph, $\Lambda_d \sim O(N \log(N))$ for large N



$$G_{E-R}\left(1000, \frac{2}{1000}\right)$$

Dan's software can express sentiments – Example of C code

```
struct module
{
    char gross[3];
    int manures[3], stools[6], sh*ts[9];
    float zap, dunk, ditch;
}
```

NOKIA